B-spline Curve Fitting: Application to Cartographic Generalization of Maritime Lines

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Abstract

Generalization is the process of abstraction applied when the scale of a map is changed. It involves modifications of data in such a way that the data can be represented in a smaller space, while best preserving geometric as well as descriptive characteristics. A map is an abstracted model representing the geometric reality. The smaller the scale, the more schematic the representation. Line cartographic generalization deals with graphic representation of lines. Many algorithms are available for an automated line cartographic generalization. Instructions for using these algorithms are often complex and representations applied ill-adapted to some generalization processes. In this paper, we explain the advantages of using B-spline curves in a line generalization process. We focus on processing of line cartographic generalization operators in a maritime context.

Keywords: Line cartographic generalization, B-spline curve.

1 INTRODUCTION

We need to distinguish between the generalization issues that are brought about by graphic representation from those which arise from modelling at different levels of spatial and semantic resolution. Second generalization can be viewed as a series of transformations (in some graphic representation of spatial information), intended to improve data legibility and understanding, and performed with respect to the interpretation which defines the end-product. These two categories have motivated research mainly in two areas: model-oriented generalization, with focus on the first stage above-mentioned, and cartographic generalization, which deals with graphic representation. Our paper is relevant to cartographic generalization.

Cartographic generalization includes the whole processings encountered when the scale of the map is changed into a smaller scale. We should produce a legible map which is as close as possible from reality. The tools currently available for automated cartographic generalization resemble those of the manual generalization. A catalogue of cartographic generalization operators has been proposed [23], including selection/elimination, aggregation, structuring, compression (or filtering), smoothing, exaggeration, caricaturing, enlargement and displacement. One can essentially distinguish between two approaches for the implementation of the working tools in generalization. One is automatic while the other is interactive. The generalization automation has been studied for over twenty years. The difficulties of providing an automatic solution points out the complexity of the problem.

The second section of the paper deals with the representations used for data modelling. Subsection 2.1 is devoted to the representation by means of a list of points. Most generalization algorithms have been developed focusing on the manipulation of vectors. Representation by means of a list of points does not provide fair modelling of curves which may have complex and varying shapes. In addition, this representation is often ill-adapted to some generalization process. In subsection 2.2, we suggest a different representation based on B-spline curves.

The third section of the paper deals with the application of B-spline representation in processing of line cartographic generalization operators in a maritime context. In subsection 3.1, we focus our attention on data compression using a bisection method on the number of control points. Line smoothing and displacement operators are developed in subsection 3.2. The strategy is based on a mechanical approach. The curve displacement is obtained through the displacement of control points. Internal and external forces are applied at control points in order to produce the desired deformation. Lastly, we introduce a technique for curve aggregation (subsection 3.3).

2 GEOMETRIC DATA MODELLING

2.1 Representation by means of a list of points

Polygonal curves are often encountered for data modelling. They are appropriate to data compression as well as to simplicity and efficiency (CPU time) of their algorithms [25].

Data compression algorithms based on polygonal curves correspond to the first generalization algorithms. Cartographer were quickly aware that cartographic results were not sufficient using these algorithms. Research in automatic generalization turned to other algorithms permitting displacements. The goals were essentially smoothing and caricaturing.

Three trends can be emphasized for smoothing. We can cite smoothing methods based on averaging, convolution or neighbouring points. Averaging techniques [3, 19] smooth small details while preserving the general shape. Algorithms considering convolution, gaussian smoothing for example [2, 18], are more regular. They are used for smoothing details which have the same size. At the opposite, smoothing algorithms based on neighbouring points [6, 27] have little influence on lines which are defined with a high density of points.

Dutton's algorithm [8] corresponds to the Brophy inverse smoothing algorithm [3]. Increasing the angularity is not a common practice in generalization. Lowe's algorithm [18] is more interesting. Tests display its interest for smoothing by limiting deviations and for caricaturing [22]. Nevertheless, the choice of the parameters is difficult.

Generalization algorithms are usually based on a representation by means of a list of points. The result quality depends on the line morphology [4]. In practice, one of the solutions often encountered consists of combining a simplification and a smoothing process: for example, Douglas and Peucker's algorithm [7] is followed by a cubic spline computation. At the present time, one should have an empiric approach for determining a solution (choice of an algorithm and its parameter values). It is natural to notice the high cost of such a technique.

More generally, this representation is not sufficient:

- for the acceptance of some resulting displayed curves,
- as a general representation method.

As regards the acceptance of some resulting displayed curves, the drawbacks of the representation by means of a list of points are linked to the broken line effects of the approximating line (Figure 2). The smaller the scale, the more angular the approximating line. This kind of representation is ill-adapted for modelling smooth shapes such as roads (Figure 1), waterways or railways. One of the solutions consists of applying continuous functions (algebraic functions, wavelets, splines).

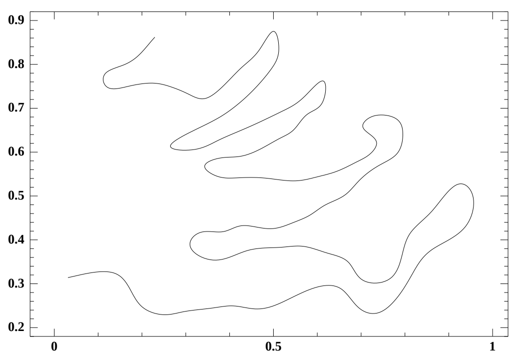

Figure 1: Initial polygonal mountain road.

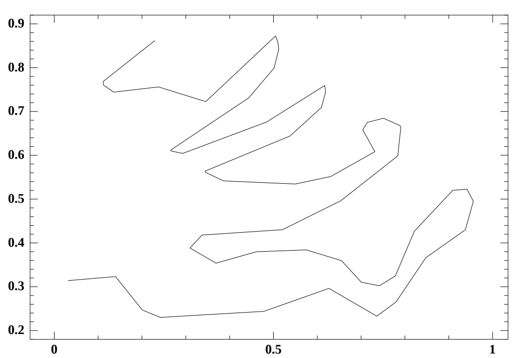

Figure 2: Jagged lines of the representation by means of a list of points.

As regards the representation method, it is limited to linear processing. For example, in order to locally modify a curvature, we should first identify the points which compose the curvature and modify them one by one afterwards. The information linked to a point is a strictly local information (each point represents a place where the line goes through), and includes no neighbourhood information [12]. Moving a point implies a discontinuous displacement along the curve. It would be useful to have a continuous displacement that is to say that shifting a point implies an automatic displacement of the neighbours. This need is important in order to satisfy the caricaturing and displacement operators (see subsection 3.2).

One should improve the generalization automation by developing other representations which lead to higher level processings. The need to introduce new representations in line cartographic generalization is detailed in Fritsch's thesis [12].

2.2 B-spline representation

B-spline curves seem suitable for the needs introduced in subsection 2.1. We assume that the reader is familiar with B-spline curves. If he is not, he can refer to [5, 9].

Our attempt is to include the fitting method in the line generalization process. Our goal is not to find a completely generic modelling but to find the best modelling for a type of lines. As B-spline curves are smooth curves, they are well-adapted for modelling smooth lines such as roads (Figure 1), railways or waterways (see also [1]). On the other hand, geographic features such as seaports should not be modelled with B-splines. The jagged line of a pier for example should remain in the generalized map (Figure 3). Polygonal algorithms ought to be applied in this case.

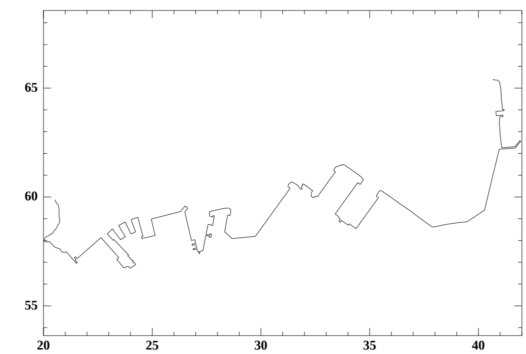

Figure 3: A type of lines adapted to polygonal algorithms.

We can essentially distinguish two approaches for the implementation of the working tools in automated generalization. One is an automatic process while the other is interactive [21]. The difficulties of providing an automatic solution have led some researchers to specialize in interactive techniques. In this case, low-level tasks are performed by the software, while high-level tasks are performed or controlled by cartographers. Through the interactive approach, B-spline representation makes it possible for a cartographer to modify a B-spline curve since additional points can easily be computed. Using local support of B-splines, a cartographer may introduce local displacements or shape modifications by first introducing additional points into the curve and by modifying them through control points (see subsection 3.2). This could be a more powerful tool (for solving line self-intersection problem for example) than the usual strategies based on a simple shifting of data points.

In addition, B-spline parameters are invariant with respect to affine transformations. As a result, B-splines are well-adapted to multiresolution problems [11, 15]. Fritsch explains in his thesis the advantage of wavelets in this context. He explains their drawbacks for solving generalization operators as well [12]. We can cite their lack of accuracy for spatial approximation.

Through the multiresolution approach, one can zoom in on a section of a curve (Figure 4) and still have a smooth representation of it using B-spline curves. As a matter of fact, one can compute additional points on the B-spline curve to improve the visual quality and the accuracy of the line (Figure 5). Such a property could be useful in embarked cartographic information systems. It is not possible for the polygonal representation to do the same: adding points in the line segments increases neither the visual quality nor the accuracy of the resulting displayed line (Figure 6).

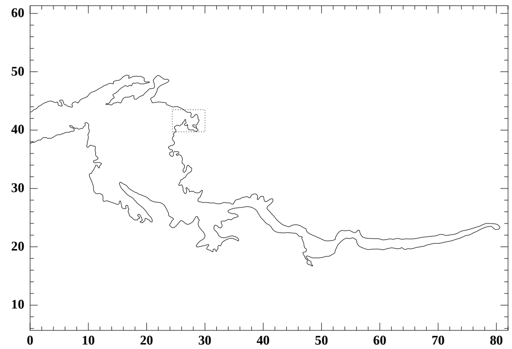

Figure 4: Initial polygonal line (1355 points).

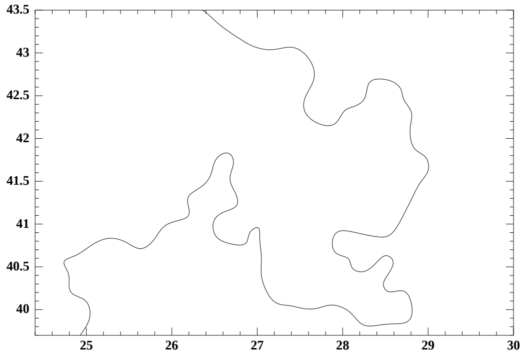

Figure 5: Interest of B-spline curves for curve section analysis.

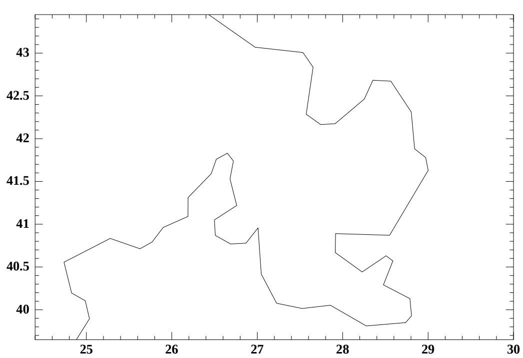

Figure 6: Drawback of piecewise linear curves for curve section analysis.

In the next section we explain how B-spline curves can be included in the cartographic generalization process by studying the generalization operators corresponding to the maritime context.

3 MARITIME LINE CARTOGRAPHIC GENERALIZATION

The main generalization operators in this context are:

- compression,
- smoothing,
- displacement,
- aggregation.

The lines we study are isobathymetric lines[1] or coastlines. The main constraint we should take into account in the modelling process is safety. Priority is to ensure safety. That is to say for coastlines, it is less dangerous for sailors if the land is shifted inside the sea than the opposite. We study the four operators in the following subsections.

[1] line whose points correspond to the same value in depth

3.1 Data compression

Let P be the polygonal curve defined by the "original" given points p_i. The general problem of data compression (or reduction) is to define a curve f with a minimal number of parameters so that $d(P, f) \leq \varepsilon$ (d being a criterion for estimating data approximation [24]). In practice, tolerance ε is chosen so that there is no visual difference between P and f for the given representation scale.

The advantage of using B-spline curves for data compression is to be able to deal with both compression and smoothing. In [25] we compare our method to some polygonal methods and spline methods (knot removal methods). Good results are obtained with respect to compression rate and computation cost.

Fitting B-splines are suitable for geographic data reduction. Data usually come from a digitizing process. This leads to digitizing errors. We assume that these are removed by a "cleaning" process. This involves the removal of spurious elements such as peaks, loops, duplicates and other redundant data. Nevertheless, noise cannot be totally removed, requiring application of fitting techniques.

We have to determine the minimum number of control points so that the corresponding B-spline approximation yields an error smaller than (or equal to) tolerance ε. A reasonable assumption is that the error in the approximating process increases as the number of control points decreases. An approximating B-spline curve is defined by solving a linear system of equations (least squares fitting technique). If we start by letting n (n being the number of the given points) the number of control points of B-spline curve f (the interpolating curve is assumed to be the curve of reference), the number of control points can then be determined applying a bisection method [25]. It may happen that a high number of points yields an initial system of non-maximum rank. In such a case, the initial number of control points is chosen slightly smaller than n.

In [25] we explain the particular choices of knot vector T, parameter values ζ_i, order k leading to fair approximation. We briefly summarize the results. Fair approximation leading to high data compression rates are obtained with:

- a uniform knot vector T,
- Hoschek's intrinsic parameterization [13],
- an order 4 (cubic B-spline curves).

In cartography, processing time should be the lowest. That is why we advise rather using a centripetal parameterization [16] than Hoschek's intrinsic parameterization. It yields a good compromise between reduction and processing time.

We have compared our strategy with polygonal compression methods such as Douglas and Peucker's algorithm [7]. The choice of this algorithm can be explained by first its interest in cartography and its ability to obtain high compression rates. The success of Douglas and Peucker's filtering algorithm in cartography may be explained by the fact that the points it selects approximate the line vertices quite well. It tends to select critical points close to those selected by humans. However, the problem is that this algorithm can create self-intersecting lines while increasing ε because no mechanism is included for discarding topologic inconsistencies. Furthermore, overlaps might result between different lines as a result of filtering each line individually (Figure 8). Results show that our method is well-adapted to smooth lines [25] by:

- improving the visual quality of the resulting displayed image by reducing without solving the self-intersecting problem (Figure 9),
- having equivalent or higher compression rates [25] (Figure 9).

In both cases, considering the algorithm with smaller tolerances may reduce the topologic errors [20, 28].

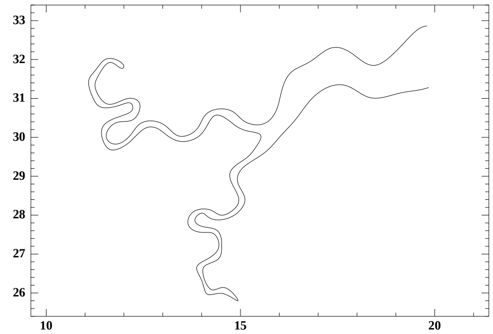

Figure 7: Initial polygonal line (981 points).

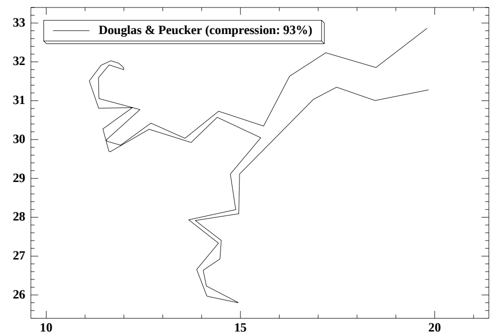

Figure 8: Approximating polygonal curve.

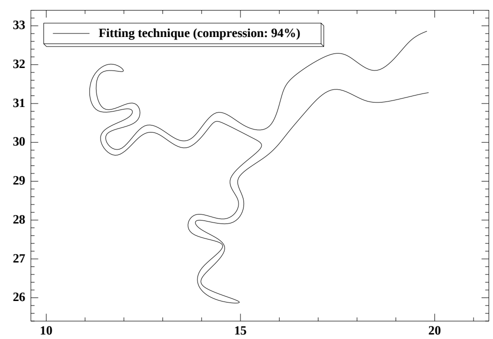

Figure 9: Approximating B-spline curve (with the same ε as in fig. 8).

Our data compression method could be considered as an elementary cartographic generalization system: results obtained by having lesser accuracy (or higher tolerance ε) lead to data compression and line smoothing (Figures 10, 11 and 12). We should now introduce additional generalization operators (displacement, exaggeration, ...) in order to improve the method. The results differ according to the applying order of the different operators [14].

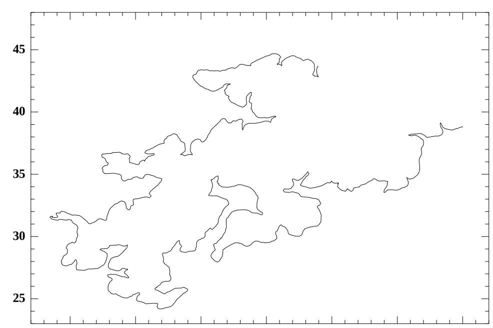

Figure 10: Initial polygonal line (1054 points).

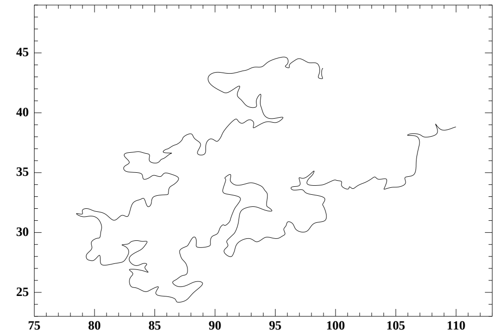

Figure 11: Generalized line obtained using our data compression algorithm while increasing ε ($\varepsilon = 0.2$mm).

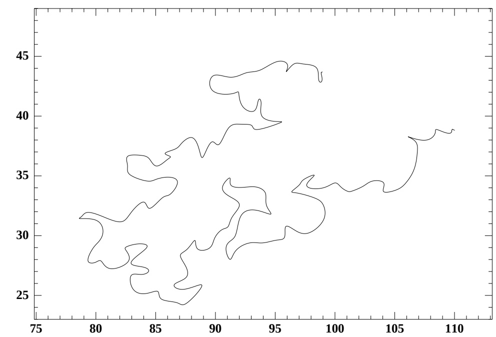

Figure 12: Generalized line obtained using our data compression algorithm while increasing ε ($\varepsilon = 0.4$mm).

3.2 Line smoothing and displacement

The goal of this section is to deal with smoothing and displacement cartographic operators. The smoothing process should preserve the vertices of the initial line. Both of them should ensure safety that is to say that each curve ought to be shifted towards deeper areas. Let us propose a rough draft of what is expected (Figure 13).

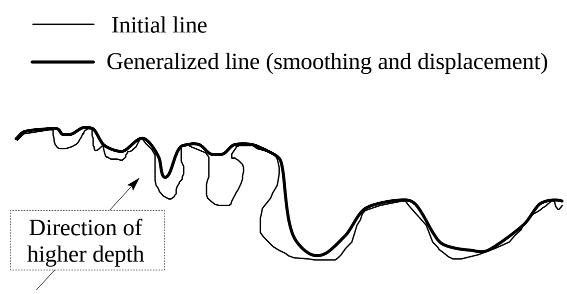

Figure 13: Manual curve smoothing and displacement.

Intuitively, one should:

- produce an approximating curve f which is as close as possible from "original" vertices (which are on the right side or safety side),
- apply forces at specific locations to shift curve f towards safety.

The first stage could be achieved applying weighted fitting techniques. These techniques are useful to locate a curve near specific points. The curve is closer to points which have higher weights. One should first select the vertices of the initial line which are well-located and impose higher weights at these vertices in the approximating problem afterwards.

We can consider Douglas and Peucker's filtering algorithm to select the vertices of the initial line. Let us remind the reader that this algorithm tends to select critical points close to those selected by a cartographer. Figure 14 is an example of vertices selected by this algorithm. Studying the angles between two consecutive line segments, we assume that a vertex is ill-located (respectively well-located) if the angle with regard to the safety constraint is smaller than (or equal to) π (respectively higher than π) (Figure 14).

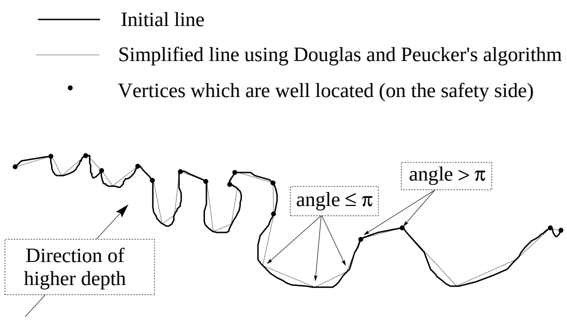

Figure 14: Selection of well-located vertices of the initial line.

The next stage is to shift approximating curve f towards safety. Usually, the modification of a model is a long and tedious process carried out through basic algorithms. These algorithms consider the well-known geometric properties between the B-spline curve and its control polygon (the line segments connecting control points). The core of the curve deformation process is the displacement of its control points. Hence, the orientation, the amplitude and the direction of each control point displacement as well as the number of control points to be moved are the unknowns of the problem. Commonly, all these parameters must be set by the user.

The method introduced by J. C. Léon and P. Trompette [17] reduces the number of parameters (controlled by the user) during the deformation process. They suggest using the analogy of the standard representation of a control polygon with a tensile cable network. The curve deformation is obtained through mechanical parameter modifications which lead to a shape modification. Each equilibrium position of the cable network (or the control polygon) can be determined solving a linear system of equations. The strategy relies on a mechanical approach permitting fast calculations as well as local and global deformations.

The mechanical model suggested by J. C. Léon and P. Trompette describes the behaviour of a network with tensile cables (or bars). They suppose there is no friction between them. Starting with an initial curve geometry, its control polygon exists and therefore an initial network is always available. The equilibrium position of the bar network depends on:

- external forces applied at control points,
- internal forces involving traction in each network's bar.

Control points are:

- either fixed control points (i.e. their coordinates are fixed): control points which are well-located in our application,
- or free control points: control points to be moved towards safety.

The determination of free or fixed control points is based on the principle of figure 14.

At that point, the geometric and mechanical problems are coupled. Curve displacement is obtained through mechanical parameter modification. The designer (or the cartographer in our context) can use two different approaches. He can:

- modify the external load field applied to the network by the addition of new forces applied to free control points (choice of these control points and choice of the direction and intensity of these new external forces),
- modify the internal force density through variations set by the designer and applied to the corresponding selected branches.

The approach proposed by J. C. Léon and P. Trompette is fully interactive and corresponds to the use of workstations having high performance graphic tools. The designer should select the area where the modification should be applied as well as the deformation mode (i.e. one among a set of pre-defined categories or libraries). This entirely determines a set of control points to be moved and their status (free or fixed). Different deformation functions are proposed in order to produce a curve stretching, shrinking, tweaking, ... The reader can refer to [17] for more details on the different behaviours of the curve and corresponding parameter values.

Such a library of functions naturally corresponds to an interactive approach of the generalization problem. Our approach of the generalization process is a semiautomatic and even automatic approach. Nevertheless, the semiautomatic technique seems to be more realistic. A minimum number of parameters for determining an initial B-spline curve (choice of parameterization ζ_i, knot vector T, order k, weights ω_i, or choice of the tolerance in the Douglas and Peucker algorithm) must be set by the user.

The (external and internal) force choice is a difficult task. One can easily compute forces in order to produce separately a curve stretching or a curve shrinking such as J. C. Léon and L. Trompette propose. On the other hand, it is more difficult to introduce accurate forces satisfying several deformations at the same time. This is the main difficulty in automated line generalization. We try to have a formula which yields an automatic internal and external force determination. The formula should take into account the geometric properties of the curve (length of the network's bars, curvature, ...). The formula we introduce (1) yields satisfactory results in most cases. It may happen that a line which has a complex geometry can create spatial conflicts. E. Fritsh suggests a technique for reducing some spatial conflicts [12]. The strategy is based on the translation of cartographic constraints into mechanical constraints. Another solution consists of applying an interactive approach (Léon-Trompette's technique, control point displacements, ...).

This leads us to explain the choice of internal and external forces for cartographic line displacement. To ensure tension in every cable, internal force densities are restricted to strictly positive values. A simple, though efficient, solution consists of setting a uniform internal force density throughout the network. Such a choice is justified if the user wants to obtain a curve deformation similar in every direction like a membrane made from an homogeneous material would behave. The results point out that we should not consider high internal force densities. Using high densities leads to reduce or even prevent the shrinking and stretching effects and to favour the fact that external forces interfere with each other. It is then more difficult to shift the curve into a pre-defined direction because of the interaction of neighbouring external forces. We suggest using slight force densities. Such a choice can reproduce the behaviour of a thin elastic curve (without return forces) being able to deal with the stretching and shrinking phenomena. However, if the user wants to differentiate the curve deformation along specific directions, he can set different force densities in the network's bars.

The location of the external forces are placed at free control points. The forces are applied according to the internal normal (i.e. the internal bisecting line). Their intensity depends on the geometry of the network's bars. Let γ_i be the internal angle between (Q_{i-1}, Q_i) and (Q_i, Q_{i+1}) line segments (Figure 15), external force density $\vec{f}_{ext_i}$ applied to free control point Q_i is determined to be:

- inversely proportional to internal angle γ_i,
- proportional to lengths d_{i-1} and d_i of line segments (Q_{i-1}, Q_i) and (Q_i, Q_{i+1}).

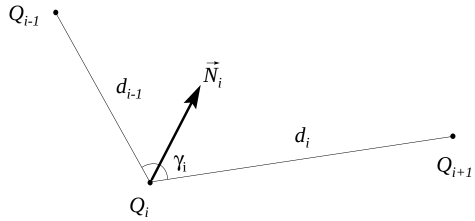

Figure 15: Bar network geometry.

The smoothing and displacement strategy described in this subsection has been implemented and extensively tested. We have tested the method with different geometric parameter choices. We can say that a formula giving satisfactory results in many cases is:

$$\vec{f}_{ext_i} = c.\frac{Min(d_{i-1}, d_i)}{\gamma_i}\vec{N}_i \quad (1)$$

where:

- $\vec{N}_i$ is the unit vector corresponding to the internal normal at control point Q_i (Figure 15),
- $Min(d_{i-1}, d_i)$ ensures that the bar which has a smaller length has a higher influence on the external force,
- c is a normalization factor,

We have analysed the results integrating them into the initial data basis. The goal is to analyse the results according to the scale of the initial data. We can say that the features of the line as well as the safety constraint are preserved. Figures 16 and 17 display the possibility of having different degrees of smoothing. We should also notice the ability of the method to smooth lines which have a complex geometry (presence of estuaries, creeks, ...) (Figure 17).

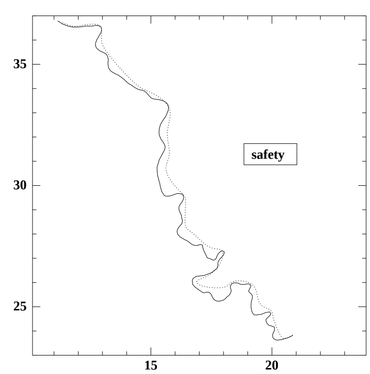

Figure 16: Curve smoothing and displacement (dotted line).

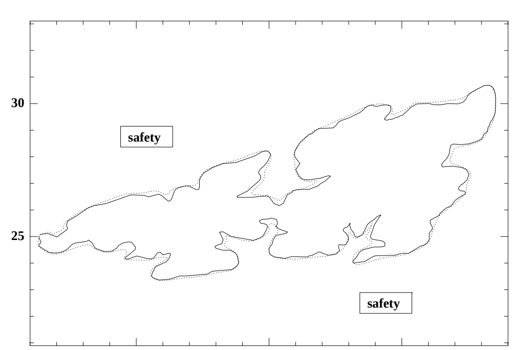

Figure 17: Curve smoothing and displacement (dotted line).

As we have to first smooth the data, we should notice that our method also realize data compression. For this smoothing, we consider Foley and Nielson's parameterization [10]. Such a choice can be explained by the ability of the method to approximate the data "in the corners" (Figure 17). This is due to the fact that the parameterization takes geometric properties of the initial line (length between the data, angles between two line segments, ...) into account. The compression rates depend on the number of control points in the B-spline curve. The number of control points is a parameter of the displacement operator. The lesser the number of control points the more schematic (smooth) and the higher compact the representation. This sort of compression differs from the compression problem described in subsection 3.1. In subsection 3.1, the problem of data reduction is to define an approximating curve with a minimal number of parameters so that there is no visual difference between the approximating and initial curves for the given representation scale.

3.3 Curve aggregation

The goal of this subsection is to propose a method for the curve aggregation process. This process consists of aggregating two curves together, at least one of them being closed. The constraints for the aggregation process are (Figure 18):

- the depth:
  - curves which have the same value in depth will be only aggregated,
  - the process should aggregate curves which are located in deeper areas.
- the closeness:
  - curves whose distance d (on the generalized map) is less than ε and which satisfy the previous constraints will be aggregated.

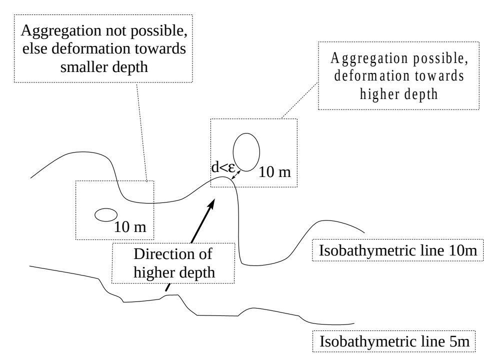

Figure 18: Constraints for the curve aggregation process.

In cartographic generalization (not only restricted to the maritime context), the merging operator is often considered. The merging process consists of merging polygons according to their spatial and semantic contexts.

Strategies which are able to merge polygons are [14]:

- the package merging method,
- the buffer method,
- the Schylberg's method [26].

The package merging method consists of determining the convex hull of initial polygons. Two main methods can be encountered to pack two convex polygons: the first one is based on a triangulation of the polygons, the other is based on a supporting line segments search. This method does not preserve the initial shape of polygons since the resulting convex hull is the convex hull of both polygons.

The dilating and eroding operators are the main operators of the buffer and Schylberg's methods. Dilating yields a stretching of the initial polygon whereas eroding yields a shrinking of the dilating polygon. Schylberg's method uses a removing operator in addition. This implies the removal of outline misrepresentations due to the application of dilating and eroding processes. The resulting polygon has less deformation than applying the buffer method. On the other hand, both methods can create "holes" inside the resulting polygon. Thus, the topology of the map is not preserved. This constitutes their main drawback.

These methods require the initial lines to be polygons. We may have open curves in our study. In addition, previous methods do not always preserve safety. That is why we have introduced a new operator. Our method is based on three stages:

- search for line segments which lean on the curve to aggregate,
- reorganization of initial points,
- approximation on this new data reorganization.

We suppose in the following paragraph that the aggregation constraints are satisfied (i.e. curves have the same depth, the process aggregates curves which are located in deeper areas, distance d between the polygonal curves is less than ε).

The first stage is based on an angle minimization problem (the orientation of the angles is taken according to the orientation of the aggregation). Starting from both extremities of the open curve, we should select point p_i on polygonal curve P and point q_i on polygon Q to aggregate so that $[p_i, q_i]$ be the first line segment obtained turning a half-line (external to Q) around point p_i. The two first line segments $[p_i, q_i]$ satisfying this condition and whose lengths are lesser than tolerance ε are the supporting line segments of the aggregation process (Figure 19). A new aggregating polygonal curve is obtained through a reorganization of the initial data of P and Q. The last stage is the determination of approximating B-spline curve f. Smoothing and displacement operators introduced in subsection 3.2 may be applied in order to shift B-spline curve f towards higher depth (Figures 20 and 21).

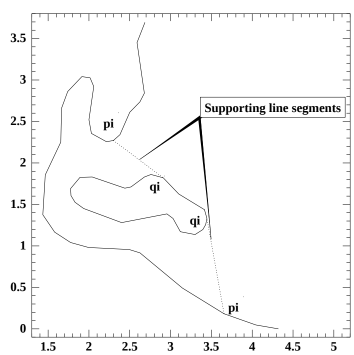

Figure 19: Supporting line segments for curve aggregation.

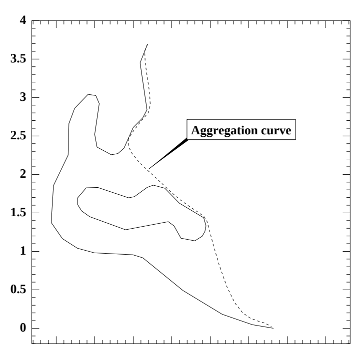

Figure 20: Curve aggregation process.

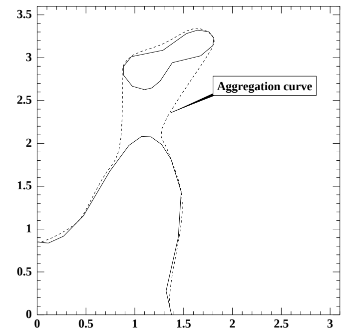

Figure 21: Curve aggregation process.

Figures 22 and 23 compare the results we obtain (dark grey curves) with results obtained from a manual aggregation (light grey curves). One can notice the similarity of the curves. Numbers in figures 22 and 23 correspond to values in depth. It should be noticed that manual curves do not always preserve safety (Figure 23).

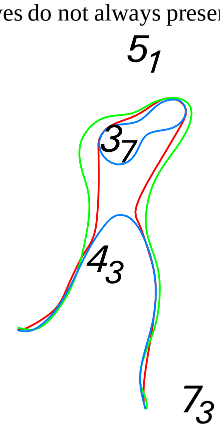

Figure 22: Comparison with a manual curve aggregation process.

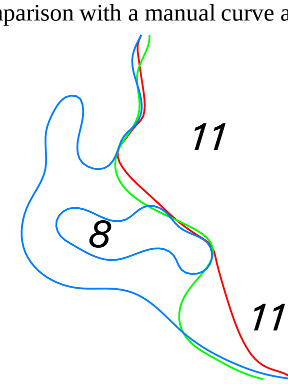

Figure 23: Comparison with a manual curve aggregation process.

4 CONCLUSION

Many researches have been focused on the creation of geographic data bases. The near future will be focused on their updating. In such a case, the generalization process is essential for producing by-products or for including data from other bases. Although many algorithms exist, there is no system which is able to produce an automatic generalization solution. This is due to the fact that the generalization method depends on the features of the initial line.

The automatic processes dealing with line cartographic generalization are difficult. There are also necessary: linear objects constitute the majority of the geographic information. This paper points out the need to introduce new representations. We suggest using B-spline curves. B-splines are introduced as an additional representation of the usual representation by means of a list of points. B-spline curves are suitable for modelling smooth objects. In this paper, we have shown that we can satisfy the main line cartographic generalization operators (compression, smoothing, displacement, aggregation). We tested them in a maritime context.

Our next goal is to introduce curvature constraints in the line generalization process. The problem is to obtain a B-spline curve with minimal curvature (to be defined). A solution would be to introduce a curvature fitting technique. We should also continue our research dealing with the other line cartographic generalization operators (caricaturing, exaggeration, ...).

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