

Название доклада:

Nonlinear Color Interpretation of Physical Processes

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Нелинейная цветовая интерпретации физических процессов

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Тезисы:

Одним из основных методов графической интерпретации на экране монитора или на бумаге мгновенного состояния или динамики развития физических процессов и систем является цветовая интерпретация. Или другими словами, формирование закона соответствия между физическими характеристиками и цветовым пространством. В данной работе предложены алгоритмы формирования таких законов соответствия для повышения информативности изображения.

В работе описаны несколько оригинальных алгоритмов формирования нелинейных законов соответствия, которые дают отличные результаты на практике. Основные подходы, которые применяются в этих алгоритмах:

- Статистический;
- Аналитический;
- Комбинация подходов.

Без ограничения общности описание алгоритмов будет проводиться на примере математических моделей, которые заданы двумерными регулярными разностными сетками.

Главная цель работы: Разработка алгоритмов повышения информативности изображений при графической интерпретации результатов расчетов физических задач и экспериментов.

Изложение сопровождается детальным сравнительным анализом традиционных методов интерпретации с предлагаемыми алгоритмами. Для наглядного представления ценности предложенных алгоритмов представлен ряд сравнительных иллюстраций. Обозначены направления дальнейших исследований методов нелинейной интерпретации при данных подходах. Отмечены недостатки каждого из предложенных алгоритмов.

Краткое содержание доклада по разделам:

В заключении определена широкая область применимости данных алгоритмов и направления исследования других подходов к данной теме, а также отмечен вопрос контроля корректности интерпретации.

Главный результат данной работы - значительное повышение информативности представления изображений при графической интерпретации результатов численного и экспериментального математического моделирования физических процессов и экспериментов.

Nonlinear Color Interpretation of Physical Processes

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Abstract

Color interpretation is one of the basic methods for graphical interpretation on screen or in paper of instantaneous state or dynamics of development of physical processes. In other words, forming a law of correspondence between physical characteristics and color space. The present paper presents the algorithms for these correspondence laws forming for enhancing the informativity comprehension.

The paper describes several original algorithms for forming nonlinear laws of correspondence, which give excellent results in practice. The basic approaches applied in these algorithms are as follows:

- A statistical one;
- An analytical one;
- Approaches combination.

Without any loss of generality the algorithms description will be performed at the example of mathematical models, which are specified with 2D regular difference grids.

General purpose of the work: Development of algorithms for enhancing pictures informativity at graphical interpretation of physical problems and experiments calculation results.

The presentation is accompanied by the detailed comparative analysis of traditional methods for interpretation with the proposed algorithms. For pictorial presentation of the proposed algorithms value the number of comparative illustrations is presented. The trends of further investigations of nonlinear interpretation methods at the specified approaches are highlighted. Imperfections of each of the proposed algorithms are mentioned.

The wide domain of applicability of the given algorithms and trends of investigation of other approaches to the given subject is determined in the conclusion and a problem about interpretation correctness control is considered as well.

The main result of the given work is a significant enhancing of image informativity at graphical interpretation of results of mathematical simulation of physical processes and experiments.

1. INTRODUCTION

Color interpretation is one of the basic methods for graphical interpretation of state and development dynamics of physical processes on a monitor screen or on the paper or, in other words, the creation of a law of correspondence between physical characteristics and color space. Different correspondence laws are used [1-3]. The given paper proposes several algorithms for forming such laws for improving the informational content of an image. One of the basic notions within this field is a notion of a color palette. Let us give a definition of a palette mentioned within this work.

Definition: A color palette is the ordered set of colors from RGB color space, which is represented with 1D array, in which each color corresponds to an integer-valued index, or in other words, its number.

Pictures 1a, 1b show different kinds of graphical presentation of palettes, the numbers mean the range of change of values of some physical characteristic.

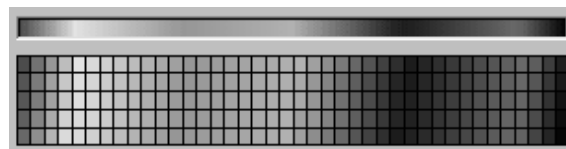


Fig 1a.

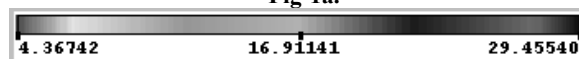


Fig 1b.

The description of the algorithms will be performed at the example of mathematical models specified by 2D regular difference grids. The given assumption does not restrict the generality of application of algorithms for other data structures, for instance for irregular grids, 3D grids, point data and others. Regular difference grids are the most frequently encountered method of continuum discretization. On the example of such grids it is more convenient to present the material. With the help of a grid and its content the approximate model of an object geometry, its substance composition and the physical state are described. Numbering within the arrays would begin with 0.

The main purpose: Development of algorithms for improving the informational content of images at color graphical interpretation of results of calculations and experiments of physical and engineering problems.

Let there be a palette of $(NC+1)$ -colors. Each color of the palette has some number. With these colors one may compose different palettes, changing a number of colors and their order within 1D array. For presenting the material we will use a palette of $(MC+1)$ -colors, the indices of these colors will be presented in some order in the 1D array $COL[MC]$, where $MC \in [0, NC]$. The given presentation is called the index presentation of colors, i. e. a color of RGB color space corresponds to each index. The derived red, green, blue colors are called the additive system of basic RGB colors mixing and it is not the only basis for color space presenting. A subtractive system of colors mixing – blue, purple and yellow colors (CMY) — is also used. As a rule, a palette is created by smooth transitions from one color to another and every user is able to choose a necessary palette depending on his taste. (Fig. 1a, 1b) However sometimes there occurs a necessity to construct a palette, which changes step-wisely.

We present the geometry of the initial data, i. e. the regular 2D grid, with 2D arrays of the coordinates $X[M, N]$, $Y[M, N]$, where $(M+1) \times (N+1)$ — is the grid dimensionality. We assume that some physical characteristics are specified in each cell. Let P denote one of the characteristics. Let us specify the corresponding 2D array for P values: $P[M-1, N-1]$. (Fig.2) $M \times N$ — is the dimensionality of this array since P is considered to be a cell characteristic. Below we give the comments concerning some other data structures.

2. DATA PREPROCESSING

For all the algorithms of color interpretation the preprocessing of P array is required. It implies the determination of a minimum and a maximum values of P value over the whole grid. Let P_{max} and P_{min} be a maximum and a minimum P values over the grid respectively.

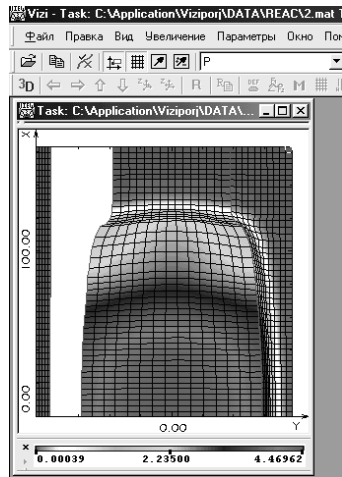


Fig 2.

3. LINEAR LAW OF CORRESPONDENCE BETWEEN THE CELLS COLOR AND A PHYSICAL CHARACTERISTICS IN THE CELLS

3.1 The algorithm description

Considering the linear law of correspondence (LLC) is important methodologically for further development of the subject and the comparative analysis. The main advantage of the given algorithm is its simplicity of implementation, absence of data preprocessing. This law is the starting point for further investigations and frequently presents good results.

LLC is the most simple and natural approach. There are two ranges of change of values, which are to be associated with each other. The first range includes changes of P value $P: I_p=[P_{max}, P_{min}]$. We assume that the segment $[P_{max}, P_{min}]$ is continuous though a set of P values is discrete. It is a natural assumption since in real, nonmathematical models or in mathematical models with a step reduction any intermediate value of P may present, therefore the odd discretization leads to a restriction of interpretation generality. The second range includes the discrete change of colors in the palette: $I_c = [0, MC]$. We do not assume the given range to be continuous. Generally speaking, nothing prevents us from considering this range to be continuous, but then we get another interpretation method, in which some nuances occur, concerned with colors occurring not form the given palette that is not always correct for visualization of mathematical results.

The task lies in associating the continuous segment I_p with the discrete integer-valued segment I_c . Each index from the segment I_c corresponds to a specific color from the color 3D space R, G, B. The concept of linear dependence of I_c on I_p is presented in Figure 3.

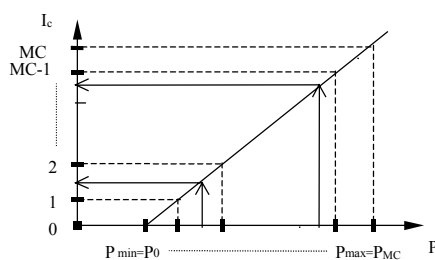


Fig 3.

We briefly describe the two variants of calculating a grid of color number in a palette for a cell over P value in this cell.

The first variant:

We calculate the coefficient of proportionality k of length of the segment I_p to the length I_c :

$$k = (P_{max} - P_{min}) / MC \quad (1)$$

k is the length of the segment of I_p decomposition with such a feature that the cells with P values from one decomposition segment will be marked with one color. Let us consider the current cell of the grid with the value $P_{ij} = P_{ij} [i, j]$, где $i \in [0, M-1]$, $j \in [0, N-1]$ and calculate the color index for it:

$I_{i,j} = (\text{int})((P_{ij} / k) + 0.5)$, (2) where (int) is marking of the whole part. This variant is equivalent to decomposition of the segment I_p into MC well-ordered intervals, each of these having the palette color corresponding to the number.

The second variant:

We compose the equation of a straight line for the correspondence law:

$$(x - 0) / (P_{max} - P_{min}) = y / MC \Rightarrow y = (x * MC) / (P_{max} - P_{min})$$

In the result we obtain the first variant but in the other way when substituting: $y = I_{i,j}$, $x = P_{ij}$

3.2 Examples of the linear law work, its imperfection and peculiarities

We consider the LLC peculiarities at the example of Figure 4, where LLC was applied.

We note that at the most cells or in other words at the greatest part of the image the change of P value is little and just in a small number of cells the value changes noticeably, therefore it is difficult to judge about the physical content of the model at its greatest part. The image results to be ill-informative. We analyze this effect. We construct the object section along AB line in Figure 4 and get the 1D graph of P value dependence on some line at the definitional domain (Fig. 5a and 5b).

Figure 6a presents the graph of P dependence on AB from the system of scientific visualization VIZI [5-7]. In Figures 5b and 6c the graph 6a and the graph of LLC are presented schematically for describing the material. Figure 5b shows that cells with the value $P \in [P_{min}, P_1]$ will be marked with color with indices 0 or 1. The rest range of the palette colors will be used for cells with the value $P \in [P_1, P_{max}]$. In Figures 5a and 5b we see that P values for a greater number of cells, which intersect the segment $[C,B]$, lie in the range $[P_{min}, P_1]$, and one color with index 0 corresponds to this segment.

Below we describe the algorithms, which correct the given imperfection of LLC. LLC algorithm works well for data with smoothly varying P characteristics on the grid. (Fig. 2)

4. STATISTICAL NONLINEAR LAW OF CORRESPONDENCE BETWEEN COLOR OF CELLS AND A PHYSICAL CHARACTERISTIC IN CELLS

4.1 General information

Statistical nonlinear law of correspondence (SLC) works well in cases similar to the ones described in Figures 4, 5. If we look at Figure 5 we will see that the most part of the grid cells have the slightly changing P value, and at the least part of the cells the abrupt change of P takes place, then we approach to the following concept. It includes statistics gathering, which correlate quantitative information with qualitative one, i. e. with P value. And then on the basis of the gathered statistics we find the colors for the cells. This algorithm is automatic, recommended for the described cases, i. e. in this case a user does not need to fit a palette, to fit a law of correspondence between I_p and I_c .

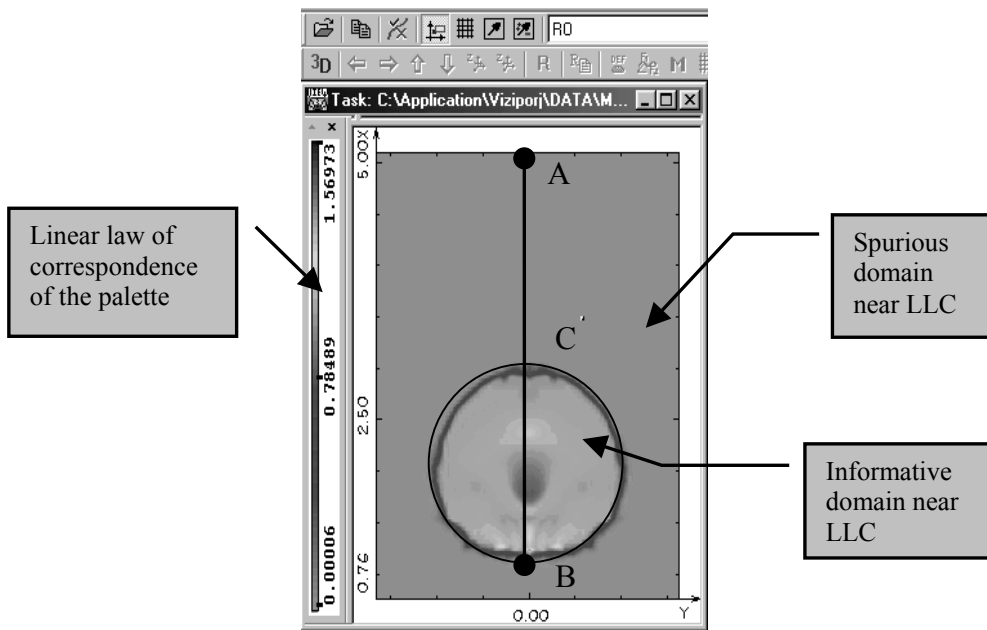


Fig 4.

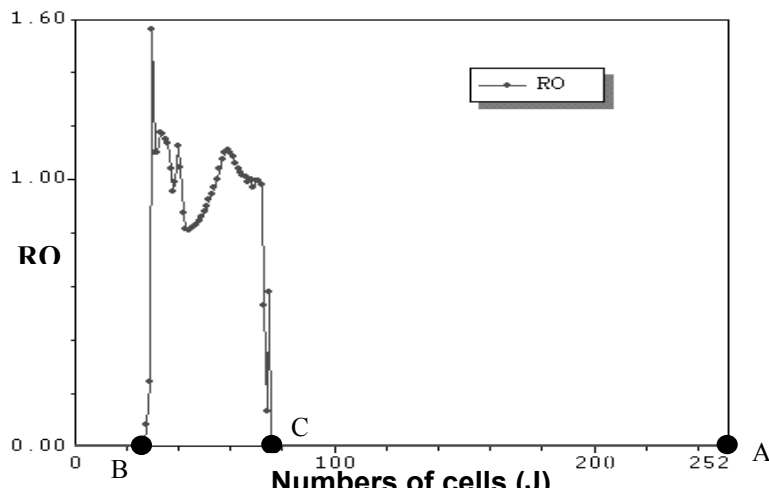


Fig 5a.

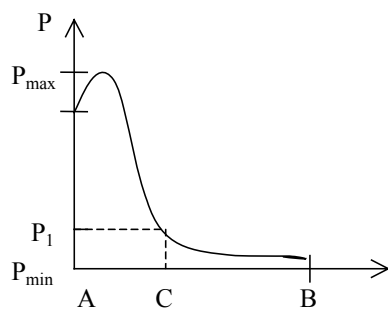


Fig 5b.

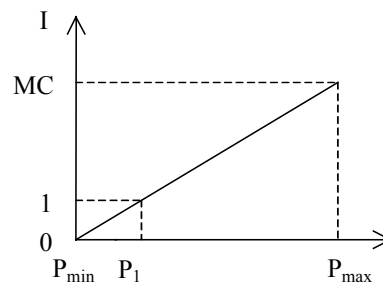


Fig 5c.

At first we describe the three algorithms for statistics gathering, from this point on the algorithm works identically for these variants. It is necessary to scan all the grid cells and fill in the statistics array STAT[MC], using LLC. At the initial initialization all the values of the array STAT equal to 0.

4.2 . The first algorithms for statistics gathering (Quantitative)

A rule for filling the cells. We consider the current cell and calculate according to LLC a color number, i. e. a number of interval, to which P value belongs to, with using formula (2).

Let $I_{p_{i,j}}$ denotes the resulting number of the color or the interval. We change a value within STAT array in a cell with the number $I_{p_{i,j}}$ in the following way:

$$STAT [I_{p_{i,j}}] = STAT [I_{p_{i,j}}] + 1.$$

And so on we perform statistics gathering over the whole grid, i. e. we calculate a number of cells belonging to each of decomposition intervals of the segment I_p . In the result we get the 1D dependence (Fig. 6) We determine the value kol – the general number of cell within the grid:

$$kol = (M+1) \times (N+1).$$

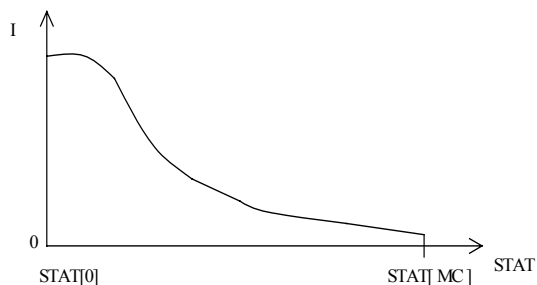


Fig 6.

We process the array STAT, performing the normalization of its values:

$$\text{STAT}[I] = \text{STAT}[I] / \text{kol}, \text{ где } I \in [0, \text{MC}].$$

For each interval of the segment I_p decomposition we obtain a portion of a number of cells, P value of which belongs to the given interval, normalized to the general number of cells. In the result STAT array will satisfy the relation:

$$\sum \text{STAT}[I] = 1.$$

Then we find a maximum and a minimum values within STAT array. Correspondingly \max_s and \min_s are these values. According to formula (1) we find the coefficient of correspondence of lengths I_p and I_c . The first algorithm of statistics gathering is finished.

4.3 The second algorithm of statistics gathering (Areal)

In 4.2 a number of cells in decomposition intervals acted as statistics criterion. In this algorithm the cells areas will be the statistics criterion. Similarly to 4.2 one is to consider the current cells and calculate according to LLC a color number or, in other words, a number of interval to which P value from the current cell belongs to. Here another method is applicable however. We divide explicitly I_p into MC intervals, i. e. we find the dividing points. In order to describe the general part of the algorithm more visual we do this after statistics gathering. Within the array REG[MC] we write down the coordinates of the dividing points:

$$\text{REG}[I] = P_{\min} + I * (P_{\max} - P_{\min}) / \text{MC} = P_{\min} + I * k, \text{ where } I \in [0, \text{MC}]$$

A rule for filling the statistics array STAT. We consider the current cell of the grid (i, j). We find a number of the interval I_p , into which a value of P_{ij} characteristics of the current cell falls.

If $\text{REG}[I] \leq P_{ij} \leq \text{REG}[I+1]$ (3), then P_{ij} belongs to an interval with number I, where $I \in [0, \text{MC}-1]$. Then we calculate the cell area according to Guron formula:

$$\text{SQR}_{ij} = 1/2 * ((x_{ij} - x_{i+1,j}) * (y_{ij} - y_{i+1,j}) + (x_{i+1,j} - x_{i+1,j+1}) * (y_{i+1,j} - y_{i+1,j+1}) + (x_{i+1,j+1} - x_{i,j+1}) * (y_{i+1,j+1} - y_{i,j+1}) + (x_{i,j+1} - x_{ij}) * (y_{i,j+1} - y_{ij})); \quad (4)$$

Then we fill in STAT array:

$$\text{STAT}[I_{P_{ij}}] = \text{STAT}[I_{P_{ij}}] + \text{SQR}_{ij}.$$

We find the total area of the grid cells (SUM_SQR):

$$\text{SUM_SQR} = \text{SUM_SQR} + \text{SQR}_{ij}.$$

Similarly to 4.2 we perform the normalization of STAT array in the following way:

$$\text{STAT}[I] = \text{STAT}[I] / \text{SUM_SQR}; \quad (5)$$

Statistics gathering is finished.

4.4 The third algorithm of statistics gathering (Divisions of the areas)

This algorithm of statistics gathering also uses the information about cells area but it is remarkable for its finer processing. In this case we need P values in the grid points not in cells. P characteristics in real problems may be specified both in cells and in points. If a value is specified in cells, then it may be interpolated into points in different ways. For example, one may transfer into a point an arithmetic average of P_{y3} from the adjacent cells over the current point.

$$\text{We obtain: } P_{y3} = (\sum P_i) / 4.$$

There are other variants for P values interpolation into points possible, for example, constructing splines or interpolating polynomials. Let P values be specified in the grid points.

A rule for statistics gathering. We consider the current cell. We calculate according to formula (4) its area: SQR_{ij} . Similar as in 4.3 we calculate the total area of all the cells SUM_SQR .

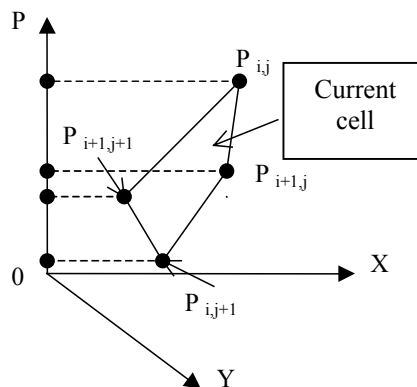


Fig 7.

The segment I_p is uniformly divided into MC intervals. Each cell point has P value. Using LLC we find a number of the interval, in which the point characteristics of P lie. In the result we get for the four points of the cell $P_p = \{P_{ij}, P_{i+1,j}, P_{i,j+1}, P_{i+1,j+1}\}$ the four numbers of color $I_p = \{I_{ij}, I_{i+1,j}, I_{i,j+1}, I_{i+1,j+1}\}$. The numbers in the general case are different, i.e. we obtain that the segment $I_0^p = [\max P_{y3}, \min P_{y3}]$ passes several intervals and the segment $I_0^c = [\max I_{y3}, \min I_{y3}]$ passes through several colors in the palette. The following technique for statistics gathering is proposed. We calculate how many intervals the segment I_0^p covers:

$$\text{NUM} = \max I_{y3} - \min I_{y3} + 1; \quad \text{NUM is a number of covered intervals.}$$

Then we divide the cell area uniformly by all the intervals with numbers I:

$$\min I_{y3} \leq I \leq \max I_{y3}.$$

We fill in STAT array:

$$\text{STAT}[I] = \text{SQR}_{ij} / \text{NUM}; \quad I \in [\max I_{y3}, \min I_{y3}].$$

And so on we analyze all the cells. It is possible to consider other ways of dividing the cells areas by the intervals (SQR), for example, the non-uniform dividing of the area by the intervals, but this method would be more complex for describing. One may perform normalization of STAT array according to formula (5).

4.5 Constructing the piecewise-linear law of correspondence for statistical data

At this step we have STAT array with the statistical information gathered above. Also we have the uniform decomposition of the segment I_p into MC intervals and MC+1 colors in the palette. At the abscissa axis we decompose the segment I_p into intervals, and at the axis of ordinates we plot off the values of STAT array for corresponding intervals. (Fig. 8) Values in STAT array are to be normalized to the range from 0 to MC, i. e. to bring in correspondence with a number of colors in the palette. Let $\max \text{STAT}[J]$ and $\min \text{STAT}[J]$ are correspondingly the maximum and the minimum values in STAT array. The normalization is performed according to the following formula:

$$\text{STAT}[I] = (\text{STAT}[I] - \min \text{STAT}[J]) / ((\max \text{STAT}[J] - \min \text{STAT}[J]) / \text{MC}); \quad (6)$$

After normalization the values of STAT belong to the range [0, MC]. In the result of all the actions we obtained the information, with which one may construct piecewise-linear, quadratic or cubic splines or interpolating polynomial.

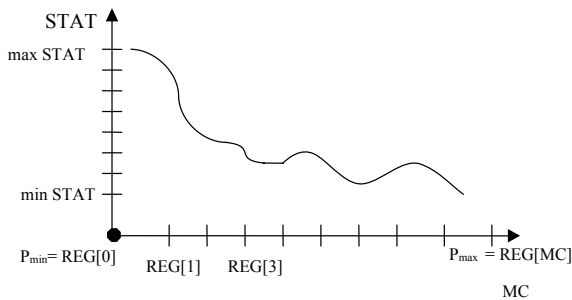


Fig 8.

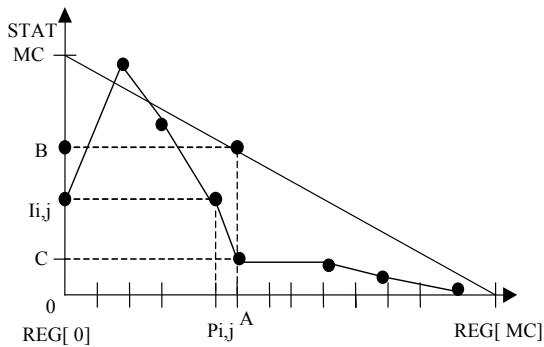


Fig 9.

We consider the process of constructing a piecewise-linear spline. In Figure 9 the points mark STAT values for each interval, joining them with segments we obtain a continuous curve, i. e. a spline of the first order. The very obtained curve will be the nonlinear statistical law of correspondence between color and a physical characteristics of P. The straight line in Figure 9 shows the corresponding LLC from 3.

Let us analyze Figure 9 before proposing formulas for calculating color by the piecewise-linear law. We consider the segment [A, REG[MC]]. In this interval the values of P exist, for which statistics is little, i. e. either there are little cells or the areas of these cells are small. When applying LLC for this segment a considerable part of the palette colors would be used, i. e. from 0 to B, and for the cells, for which statistics is large, i. e. their values lie in the range [REG[0], A], a smaller part of the palette from B to MC would correspond. This leads to reducing the image informational content. When using the statistical law we get that for the cells with little statistics and with P values from the range [A, REG[MC]] a small number of colors from 0 to C would correspond, and for the cells with large statistics and with P values from the range [REG[0], A] the rest part of the palette range from C to MC would be used. Frequently encountered is the situation when at large statistics there are small changes of P value. It means that at the given statistical law of correspondence the small changes of P are as a rule well presented, and the larger ones disappear.

Now we give formulas for calculating a number of colors in the palette for the cells using the formulas of the spline of the first order. We consider the current cell with the value P_{ij} . We determine by LLC a number of the interval I of the segment I_p , in which the value P_{ij} lies. At LLC the color with the number I_{ij} corresponds to the cell. And at SLC the value P_{ij} are to be substitute into the formula of the spline:

$$I_{ij} = \text{STAT} [I] * ((\text{REG} [I + 1] - P_{ij}) / k) + \text{STAT} [I + 1] * ((P_{ij} - \text{REG} [I]) / k); (7)$$

k is taken from formula (1).

Figure 10 presents an example of SLC work for the same model as in Figure 4.

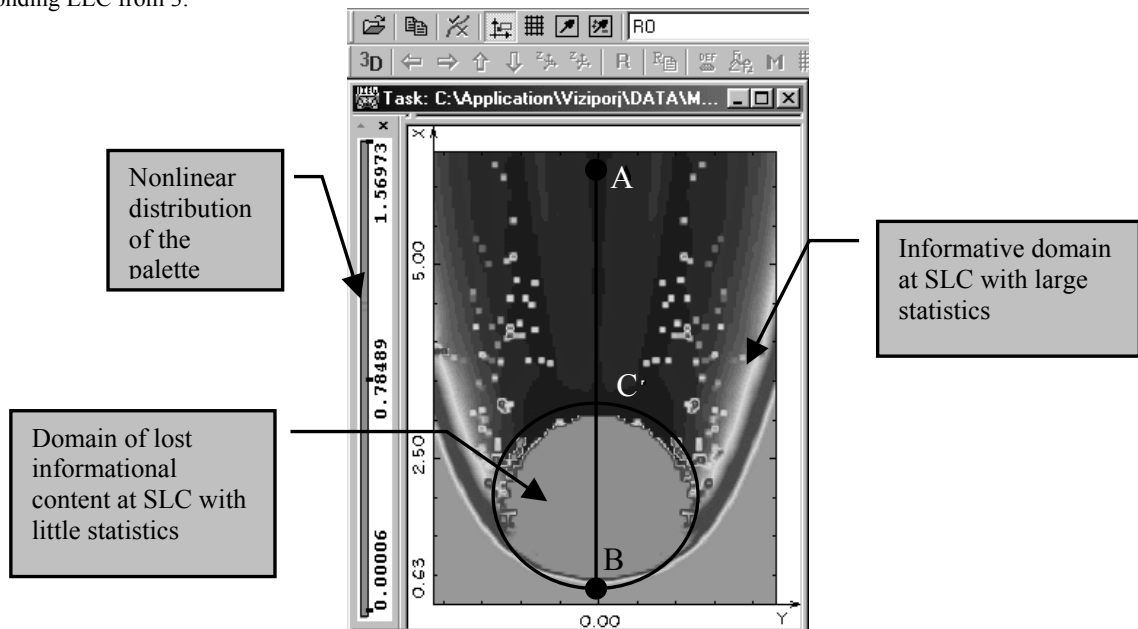


Fig 10.

4.6 Methods of SLC evaluating

Note 1: Applying some other methods of statistical information gathering. We give several examples without analyzing them:

- One may consider not a scalar value P, but its gradient or its change on the grid;
- One may combine the results of statistics gathered by different methods.

Note 2: For various data the individual fitting of a method of statistics gathering is required.

Note 3: After statistics gathering one may apply different interpolating methods, piecewise-linear, quadratic or cubic splines and interpolating polynomials of Lagrange or Chebyshev [4].

Note 4: In 4.4 we used the uniform dividing the cell area into a number of intervals, which cover the segment $[\max P_{ij}, \min P_{ij}]$. One may apply non-uniform more precise dividing of the area for statistics gathering.

Note 5: Generally speaking, P value is a function of many variables, therefore one may propose statistics gathering by some set of 1D model sections.

Main feature of SLC: Statistics gathering allow reducing the multidimensional distribution of P scalar value to 1D dependence of the palette colors at color interpretation of P value.

The good properties are:

- The way of how to improve the image information content becomes clearer;
- The palette is also a 1D array that allows presenting the correspondence between P and the palette visually and simply;
- There is a possibility to apply 1D splines and interpolating polynomials.

2. In 4.5 we noted that the informational content for the image disappears for the range P with little statistics. Below in 4.8 we give a method for eliminating this imperfection. (Fig. 10)
3. The given SLC gives a bad result in cases when P value is distributed over the grid in some measure uniformly, without big sudden changes or when the grid is of very small dimensionality. Figure 11a presents an example of such an image. Figures 11b and 11c present the sketchy graphs of typical sections and Figure 11d presents the graph of such real section for the model in Figure 11a. This is due to the fact that in the process of statistics gathering there occur dips or peaks in some intervals, i. e. there is no statistics in them because of lack of information or because it is unreasonably large. (Fig. 12)

4.7 Imperfections

1. Some additional time and memory resources are required for statistics gathering.

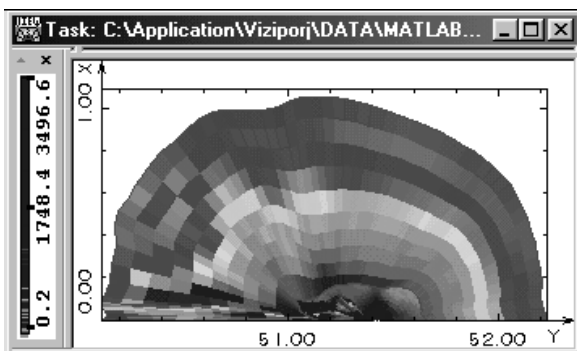


Fig 11a.

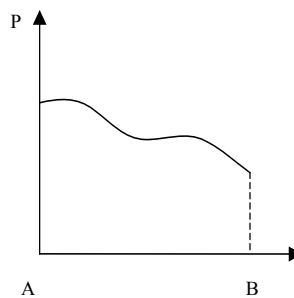


Fig 11b.

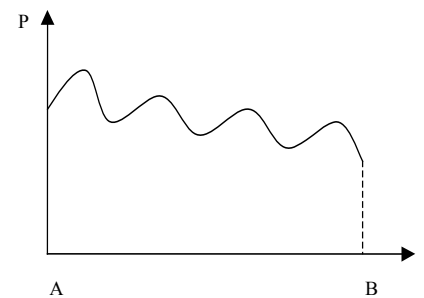


Fig 11c.

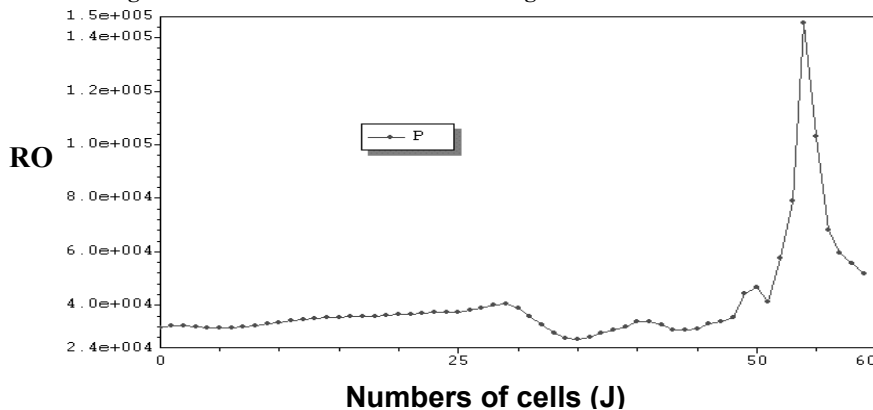


Fig 11d

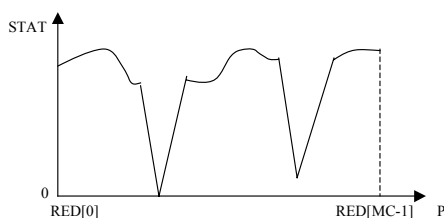


Fig 12.

The reason of this lies small changes of P value and/or great discreteness of the grid. By the algorithm of statistics gathering the dips are also included into the resulting spline, that leads to abrupt

changes of colors and this does not confirm with the real situation.

This imperfection may be eliminated in the different ways:

The first method: To use different laws of correspondence, for example, LLC.

The second method: To use a different algorithm for statistics gathering.

The third method: Statistics correction implies eliminating of dips or “noises”. It means the elimination of non-informative intervals of the segment I_p decomposition or their integration.

We consider Figure 13. The dip occurred in the third interval. It is necessary to take away the value for this interval out of STAT array, i. e. the value STAT[2] is not to be taken into account when constructing the spline. The graph would change, as it is shown in Figure 13 with the dotted line.

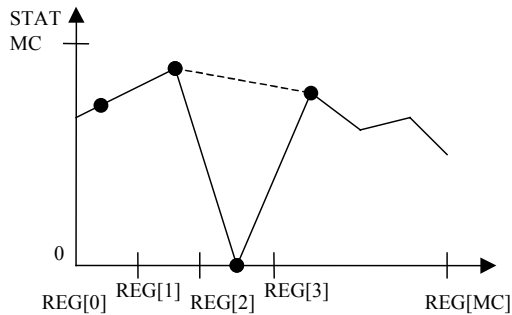


Fig 13.

4. Colors confusion. It means that different P values may correspond to one color in the palette. It takes place in places where the derivative of SLC graph changes its sign. The comments are as follows:

- Color interpretation refers to the qualitative data analysis and to the trending, therefore together with the indices in informational messages this imperfection is not of great significance;
- One may propose however the methods for its total elimination. One may, for example, apply different palettes at the ranges of the derivative constant value.

5. ALGORITHM FOR SLC CORRECTION WITH CONSIDERING LLC

Then the value I_{pe3} is to be entered into STAT array: $STAT[I_n] = I_{pe3}$. Then one may normalize the values within STAT array in such a way that all the colors within the palette would be included, i. e. from 0 to MC according to formula (6).

This algorithm (SLC+LLC) allows eliminating the mutual imperfection of linear and statistical laws of correspondence. Sections 3.2 and 4.7 and Figures 4 and 10 noted the imperfections of these laws. There occurs a concept of integrating these two algorithms. It means the attempt to eliminate the imperfection of LLC with the help of the statistical law, i. e. to present small changes of P value qualitatively in colors, and on the other hand with the help of the linear law to keep good presentation of P intervals with little statistics, i. e. where P changes very rapidly.

Below we give the two methods for merging these laws.

Method 1: One should determine the intervals for the segment I_p , in which we will apply the linear law and in which — the nonlinear law. For example, in Figure 9 in the segment [REG[0], A] it is expediently to apply the SLC algorithm, and in the segment [A, REG[MC]] it is expediently to apply LLC. This method has one weak point, i. e. fitting the points of a law changing. It should be noted that such a point may not be a single one. The comments are as follows:

- One may consider algorithms, which automatically fit these points. But all the criteria for finding the points of a law changing would have partly an empirical or approximate character.
- One may apply a method of manual points fitting, it is a visual-empirical method. It that an engineer, while looking at the statistics graph of P or P of conditional 1D argument, puts or enters interactively the dividing points of the segment I_p .

Method 2 (SLC+LLC): The concept of this method lies in automatic joining the two laws of correspondence. Assume that statistics has been gathered by some method, i. e. we have the normalized array STAT. We consider the current cell. We calculate for it a number of color by LLC (2). We denote it as I_n . Further we

calculate for it a number of color by SLC (8). We denote it as I_{kn} . We form the resulting color number I_{pe3} :

$$I_{pe3} = (I_n + I_{kn}) / 2;$$

We have obtained the average arithmetic for the cell color. Some other ways of joining the numbers of I_n and I_{kn} are also possible. We consider the example with a graph (Fig. 14) Graphically it means that at the graph in Figure 14 we enlarge the amplitude. (The dotted line).

The given method gives excellent results in practice. See Figure 15.

6. OTHER METHODS OF FORMING NONLINEAR LAWS OF CORRESPONDENCE BETWEEN COLOR AND PHYSICAL CHARACTERISTICS

This chapter is briefly deals with two algorithms for forming nonlinear laws, which are sometimes used in graphical systems. Here we describe some peculiarities and improvements for them.

This algorithm combines automaticity and interactivity of constructing the nonlinear laws of correspondence. Its concept lies in applying standard analytical functions or their algebraic combinations. There is an infinite set of these functions, so it is natural that one should fit its well-founded set and determine variable parameters in analytical formulas, which one may continuously change and obtain a new quality of image.

It is natural to consider the simplest functions: e^x , $1/e^x$, $\ln x$, $\log_a x$, ax^α and others. Without preprocessing one typically can not apply analytical expressions, since each specific function has its own definitional domain, i. e. a strictly defined geometrical place for potential arguments, a specific curvature shape of a graph.

We consider the peculiarity of this method at the example of the exponential function: $f(x) = e^x$. In Figure 16 we consider its graph and, for example, the object section profile. We can see that the function $f(x)$ should be somehow transformed.

Let 0 corresponds to point A, if it does not, then one can always perform a parallel shift, and to displace the interval [A, B] to [B-A]. In the graph $y = e^x$ we are interested in its very part, where the bend takes place, for example, the interval [C, D]. Adjusting this bend we would obtain different informational content of the image and moving it along the abscissa axis we would obtain different correspondence laws.

We analyze this graph. We can see that the greater part of the segment [A, E] lies in a small interval of P values: $[P_{min}, P_0]$ — it means that at the linear law of distribution the greater part of the image would be marked with close colors or even with one color. It is necessary to transform the function $y = e^x$ to the required form. We consider the reverse graph, where P lies in the abscissa axis and in the axis of ordinate we plot off the palette colors. X has a meaning of P value.

Into the interval $[0, P_0]$ one color transfers. It is necessary to attain that several colors would correspond to the interval $[0, P_0]$. It is obvious that the graph e^{op} is to be drawn down for a unity in order to hold all the values of P or to transfer far to the right. “Far” means that e^{op} can never be equal to 0 at $\forall \alpha$ and $\forall x$. Therefore the transference to the right is not very convenient since it does not matter how far the graph would be transferred to the right, the value of P out of some vicinity P_{min} are anyway indefinite. Let us process this function. We perform the transfer:

$$y = f(x) = e^{\alpha x} - 1.$$

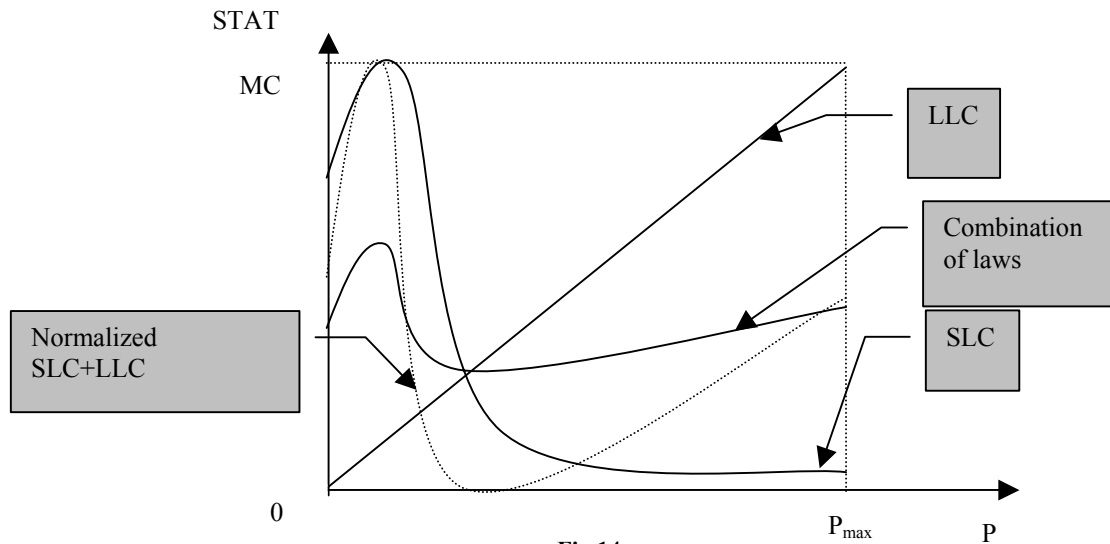


Fig 14.

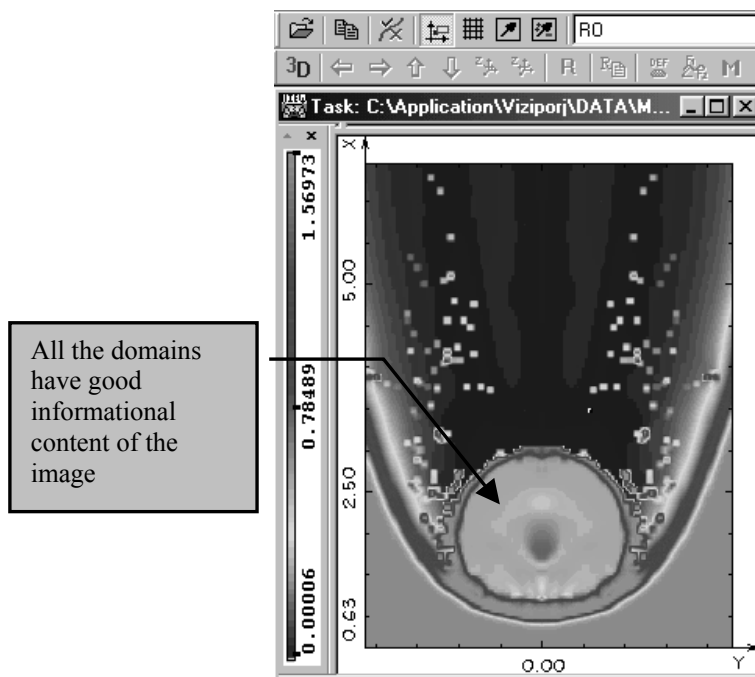


Fig 15.

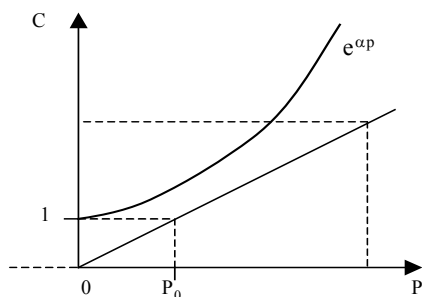


Fig 16.

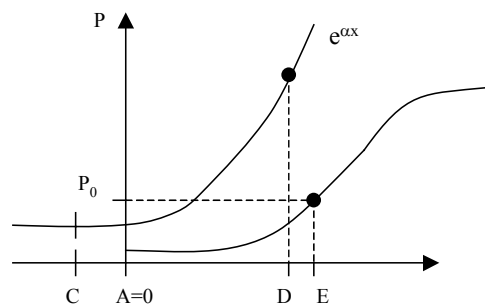


Fig 17.

In the result the graph would pass through the origin of coordinates, i. e. one end of the distribution graph is fixed in the point $(P_{\min}, 0)$ or as far as for the given case in the point $(0, 0)$. Now one must determine the ordinate value at $x = P_{\max}$, i. e. $y = e^{\alpha P_{\max}}$. By the ordinates axis we get the range $[0, e^{\alpha P_{\max}}]$. Then we divide this range into MC parts, i. e. by a number of colors in the palette. As a matter of fact we have obtained the nonlinear law of colors distribution by the function $y = e^{\alpha x}$, where

$\alpha \in \mathbb{R} \setminus \{\emptyset\}$. At $\alpha = \emptyset$ the law has no sense. In order to get the color of a specific cell we come up to the formula:

$$I_{ij} = e^{\alpha P_{ij}} / (e^{\alpha P_{\max}} / MC).$$

The example of work of exponential law and the reverse exponential law are given in Figures 18-21. Figure 18 shows that the exponential law at $\alpha = 9$ gives a bad result over the whole image. Figure 19 shows that the exponential law at $\alpha = 50$ gives a good result at great changes of a characteristics. Figures 20 and 21 show that the law of the reverse exponent at $\alpha = 9$ and $\alpha = 50$ give

good results at small changes of a characteristics. We may be not satisfied with the result because of not strong non-linearity. To change the result one may apply the several ways:

- Changing α parameter;

- Parallel transfer of the function graph along the abscissa axis;
- Applying the law of the reverse exponent;
- Applying the combination of the exponential law and the law of the reverse exponent.

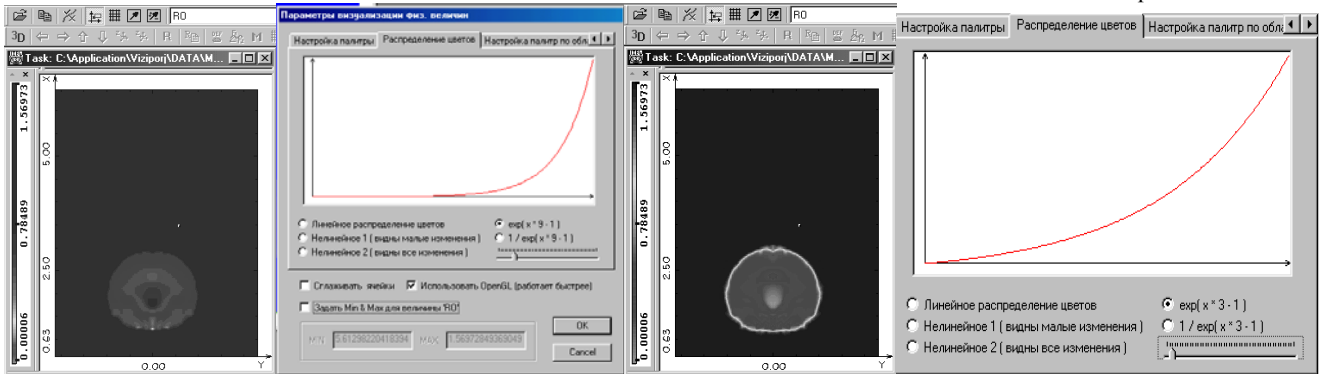


Fig 18a.

Fig 18b.

Fig 19a

Fig 19b.

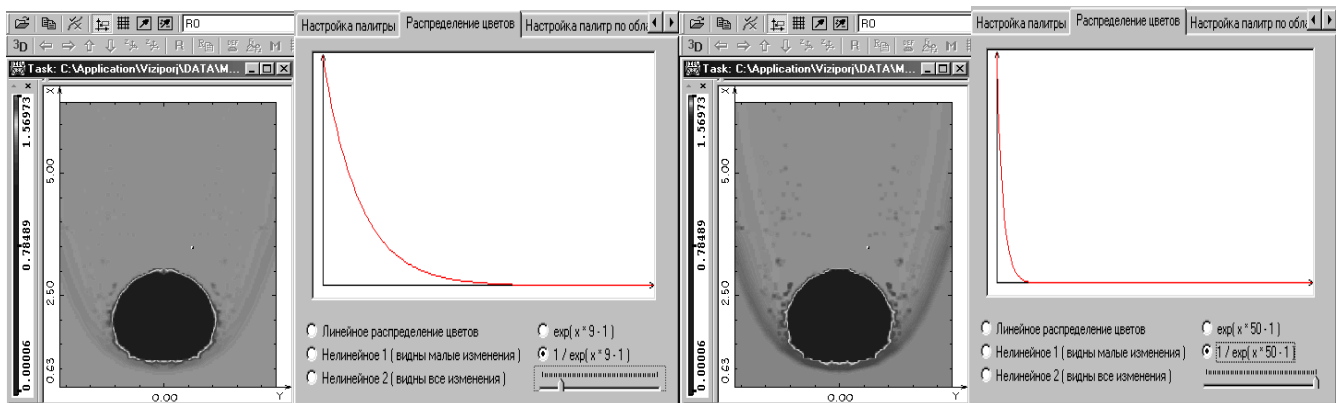


Fig 20a.

Fig 20b.

Fig 21a.

Fig 21b.

7. CONCLUSION

In the conclusion we note that statistical laws and the law of the reverse exponent with parameters are successfully implemented in the system of scientific visualization "VIZI2D". [5-8] All the illustrations from VIZI were presented above at one mathematical model for better comparative analysis. Below in Figures 22 – 25 the results for another model are presented.

The main result: Considerable improvement of informational content of objects at graphical interpretation of results of mathematical simulation of physical processes.

Summarizing: In the given paper the algorithms were stated at the example of 2D regular grid, however these algorithms may be applied without changes for some other data structures, for example:

- 3D regular grids;
- Irregular grids;
- Gridless models.

Further investigations of nonlinear laws of correspondence continue along the following trends:

- Other methods of statistics gathering;
- Other principles of laws forming, for example, a palette fitting;
- A detailed investigation of analytical laws;
- Methods of estimation of an object informational content or measures of information amount within an image.[9]

8. REFERENCES

1. Bulatov V., Dmitriev V. How to See the Invisible. Part 1 - "Computer Press", N 4, 1993.
2. Dmitriev V. How to See the Invisible. Part 2 - "Computer Press", N 8, 1994.
3. D. Rogers. Algorithmic Bases of Computer Graphics. - M., Mir, 1989.
4. Bakhvalov N.S. Numerical Methods. Moscow. Nauka. 1973.
5. Kryukov V. M., Mogilenskikh D.V., Fedorov V. V. 2D and 3D Visualization of Results of Mathematical Simulation of Complex Physical Systems. Graphicon'95. Proceedings of Conference. V 2. St. Petersburg. July 3 – 7, 1995. The Joint International Mathematical Conference. Proceedings of Conference. May 20-25, 1996. Snezhinsk.
6. Mogilenskikh D.V., Fedorov V. V., Pavlov I. V. Systems of Scientific Visualization "VIZI" for Graphical Representation of Results of Mathematical Simulation. The Third Siberian Congress on Applied Industrial Mathematics (INPRIM-98). Abstracts. Part III. Novosibirsk. June 22-27, 1998. Published by the Institute of Mathematics of SB RAS.
7. Mogilenskikh D.V., Fedorov V. V., Pavlov I. V. Visualization of 2D Results of Numerical Simulation of Physical Processes. "VIZI" Visualization System in OS Windows. V Zababakhin Scientific Talks. The International Conference, September 21-25, 1998. Snezhinsk. Pp. 134,135.
8. Sh.-K. Chan. Principles for Designing the Systems of Visual Information. M., Mir, 1994.

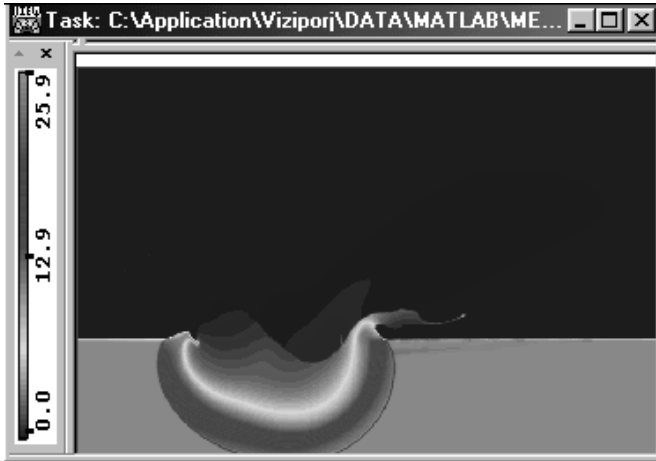


Fig 22: (LLC).

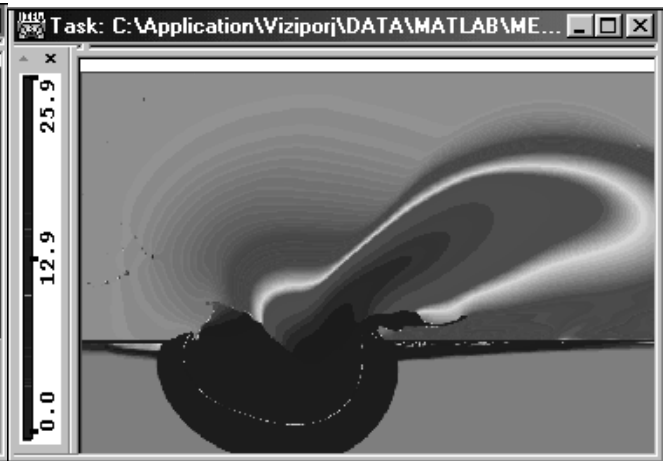


Fig 23: (SLC).

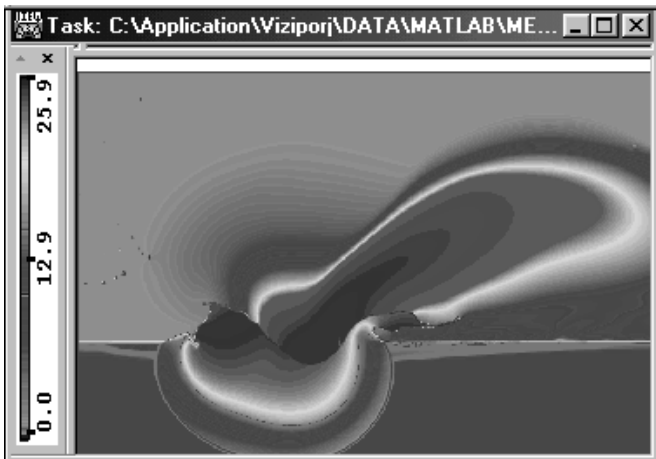


Fig 24: (LLC+SLC).

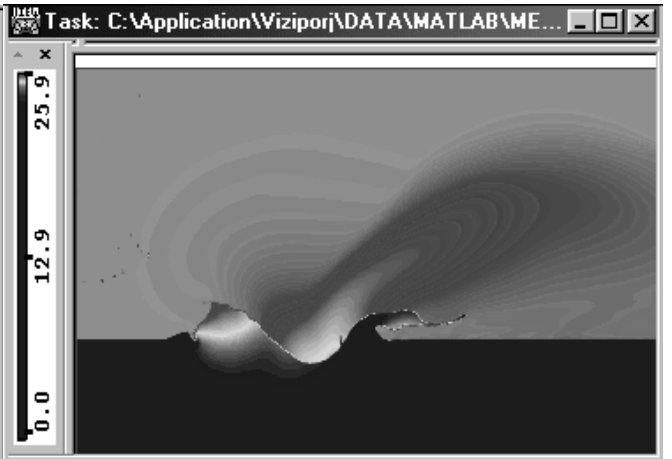


Fig 25: (Law of reverse exponent).