A Reverse Scheme For Quadrilateral Meshes

Yacine Boumzaid* LE2I, Universite de Bourgogne

Sandrine Lanquetin† LE2I, Universite de Bourgogne

Marc Neveu ‡ LE2I, Universite de Bourgogne

Abstract

Reverse subdivision constructs a coarse mesh of a model from a finer mesh of this same model. In [1] Lanquetin and Neveu propose a reverse mask for the Catmull-Clark scheme which consists in locally reversing Catmull-Clark original formula for even control points, but this mask can not be applied in reversing other variants such as Quad-averaging scheme of Warren and Weimer [2]. In this paper, we derive a reverse mask for Catmull-Clark. This mask is parameterized and can also be used for reversing other quad schemes as Quad-averaging scheme.

Keywords: *Reverse Subdivision, multiresolution, Catmull-Clark scheme, Reverse loop Subdivision, Reverse Catmull-Clark Subdivision.*

1. INTRODUCTION

Since their first appearance in 1978, subdivision algorithms for generating surfaces of arbitrary topology have gained widespread popularity in computer graphics and are being evaluated in engineering applications. This development was complemented by ongoing efforts to develop appropriate mathematical tools for a thorough analysis, and today, many of the fascinating properties of subdivision are well understood. Since the earliest subdivision surfaces in 1978, many subdivision schemes were proposed. Some are approximating as Catmull-Clark [3], Loop [4] and Doo-Sabin [5], and others are interpolating as Kobbelt [6], Butterfly [7]. Moreover subdivision surfaces are more and more used in CAGD, and in this field most meshes are quadrilateral, in coherence with parametric surfaces (Bezier, B-splines, NURBS). Subdivision methods produce a sequence of increasingly fine meshes. On the contrary, it can be interesting to go quickly from a mesh to a coarser one. Using a local formula for decreasing the resolution of a mesh is a crucial element for the implementation of multiresolution surfaces. It reverses the subdivision process. While formulas for subdividing meshes are local, the existence of local formulas for the respective reverse subdivision is less evident. Using a local formula to reverse the subdivision process implies that the local connectivity of a mesh is clearly identified. For instance triangular structures, quadrilateral schemes can be detected following taubin's work [8].

There are several global methods such as multiresolution methods [9], as Loop reverse subdivision [10]. Local methods for reverse subdivision are of interest because they use small neighborhoods around a vertex (with reduced topological information) and induce small systems to solve. Samavati and Bartels determined the local reverse subdivision masks for the Butterfly and Loop scheme restricted to regular vertices (valence 6) in [11]. Samavati et al. focused on the Doo-Sabin scheme for arbitrary meshes in [12]. Samavati, et al. [13] propose a local method for the Loop scheme which consists in locally reversing the formula for a given set of vertices. Lanquetin and Neveu [1] propose a reverse mask of Catmull-Clark scheme which consists in locally reversing the Catmull-Clark original formula for even control points. The Catmull-Clark formula for even control points can be chosen in different ways. In this paper, we use the simplest one, to develop a local method to reverse Catmull-Clark and Quad-averaging subdivisions. Section 2 overviews the Catmull-Clark scheme, Quad-averaging scheme and the reverse mask for Catmull-Clark found in [1]. Then we present our method for reversing Catmull-Clark and Quad-averaging schemes in Section 3. Results are shown in Section 4, illustrating different cases that can occur.

2. BACKGROUND

Subdivision is a repetitive refinement process that gradually converts a given coarse mesh to finer meshes to generate a smooth surface at the limit. An arbitrary mesh M can be denoted by the pair (F, V), where F shows the faces of M, and V denotes the vertices of M. Each element $v \in V$ has the spatial coordinates, (x, y, z), and each element $f \in F$ is assigned a list that includes all indices of its adjacent vertices in V. Catmull-Clark, Doo-Sabin, Butterfly and Loop subdivisions are some important cases . The input for subdivision methods is $M^0 = (F^0, V^0)$ a control mesh. In each step of a subdivision method, the mesh $M^k = (F^k, V^k)$ is converted to a new and finer mesh $M^{k+1} = (F^{k+1}, V^{k+1})$. This conversion is done through some local affine operations on V^k, together with a mapping process from the faces of F^k to those of F^{k+1}. The affine operations are usually described by masks, or matrices, that are smoothing filters. Consequently, by successively applying a subdivision method, a hierarchy $M^0, M^1, M^2, ..., M^k, \ . \ . \ .$ is obtained that usually converges to a smooth surface.

2.1 The Catmull-Clark Subdivision Scheme

In computer graphics, the Catmull-Clark algorithm is used in subdivision surface modeling to create smooth surfaces. It was devised by Edwin Catmull and Jim Clark in 1978 as a generalization of bicubic uniform B-spline surfaces to arbitrary topology. Catmull-Clark surfaces are defined recursively, using the following refinement scheme: Start with a mesh of an arbitrary polyhedron $M^k = (F^k, V^k)$. All the vertices in the mesh shall be called original points. In each step of subdivision, the mesh $M^k = (F^k, V^k)$ is converted to a new and finer mesh $M^{k+1} = (F^{k+1}, V^{k+1})$. The set of new vertices V^{k+1} includes three types of vertices (Figure 1):

• A face control point (f_i^{k+1}) for an n-gon is computed as the average of the corners of the polygon:

$$f_i^{k+1} = \frac{1}{N} \sum_{i=1}^{N} v_i^k \tag{1}$$

where N is the number of corners.

• An edge control point (e_i^{k+1}) is the average of the endpoints of the edge and newly computed face control points of adjacent faces:

$$e_i^{k+1} = \frac{1}{4}(v^k + v_i^k + f_i^{k+1} + f_{i+1}^{k+1}). \tag{2}$$

• An even control point (v^{k+1}) is a weighted average of its incident

*Yacine.Boumzaid@u-bourgogne.fr

†Sandrine.Lanquetin@u-bourgogne.fr

‡Marc.Neveu@u-bourgogne.fr

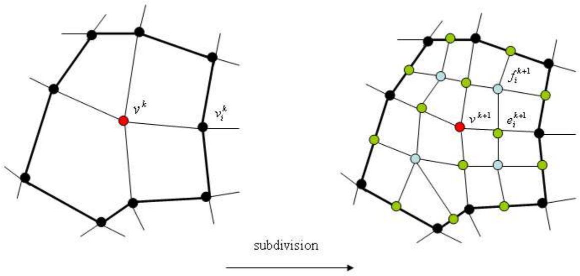

Figure 1: Catmull-Clark subdivision.

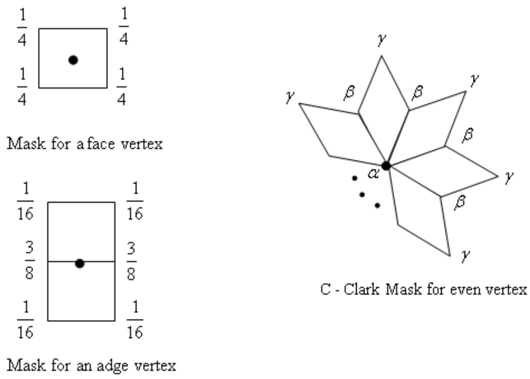

Figure 3: General case for Catmull-Clark masks: $\beta = \frac{3}{2n^2}$, $\gamma = \frac{1}{4n^2}$, $\alpha = 1 - n(\beta + \gamma)$.

vertices of the same level and of the face points of the incident faces:

$$v^{k+1} = \frac{n-2}{n} v^k + \frac{1}{n^2}\sum_{i=1}^{n} e_i^k + \frac{1}{n^2}\sum_{i=1}^{n} f_i^{k+1} \tag{3}$$

The new mesh will consist only of quadrilaterals, which won't in general be planar. The new mesh will generally look smoother than the old mesh. Repeated subdivision results in smoother meshes. It can be shown that the limit surface obtained by this refinement process is C^2 at ordinary vertices and C^1 everywhere else. If the rules of Catmull-Clark scheme are defined for meshes with quadrilateral faces, then in each step of subdivision, each face in F^k is replaced by four new quads that become the faces of F^{k+1} (Figure 2). In this case the masks are shown in Figure3.

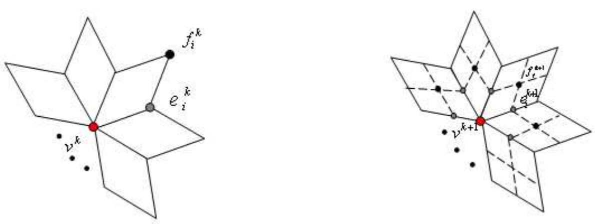

Figure 2: Situation around a vertex v^k before and after subdivision.

Let $e_1^k, e_2^k, ...e_n^k$, $f_1^k, ...f_n^k$be the set of neighbors of v^k in M^k. In addition, let v^{k+1} be the even vertex of v^k, and $e_1^{k+1}, e_2^{k+1}, ...e_n^{k+1}$, $f_1^{k+1}, ...f_n^{k+1}$ be the corresponding edge vertices and face vertices (n is the valence). The edge vertices e_i^{k+1}, face vertices f_i^{k+1} and even vertices v^{k+1} defined by equations (1), (2) and (3)become:

The edge vertex :

$$e_i^{k+1} = \frac{3}{8} v^k + \frac{3}{8} e_i^k + \frac{1}{16} f_i^k + \frac{1}{16} e_{i+1}^k + \frac{1}{16} e_{i-1}^k + \frac{1}{16} f_{i-1}^k \tag{4}$$

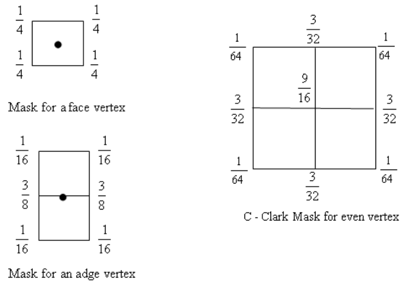

Figure 4: Catmull-Clark masks for a regular vertex

The face vertex :

$$f_i^{k+1} = \frac{1}{4} v^k + \frac{1}{4} e_i^k + \frac{1}{4} f_i^k + \frac{1}{4} e_{i+1}^k \tag{5}$$

The position of even vertex:

$$v^{k+1} = \alpha v^k + \beta \sum_{i=1}^{n} e_i^k + \gamma \sum_{i=1}^{n} f_i^k \tag{6}$$

With: $\alpha = 1 - n(\beta + \gamma)$, $\beta = \frac{3}{2n^2}$, $\gamma = \frac{1}{4n^2}$

2.2 Quad-Averaging scheme

The Quad-averaging scheme was described by Warren and Weimer in [2]. It can produce a smooth surface but not necessarily at extraordinary vertices. To apply these subdivision rules to an arbitrary quad mesh, we need only to generalize the vertex rule from the valence four case (Figure 4) to vertices of arbitrary valence. In the regular case, the mask can be decomposed into the sum of four sub-masks:

$$\begin{pmatrix} \frac{1}{64} & \frac{3}{32} & \frac{1}{64} \\ \frac{3}{32} & \frac{9}{16} & \frac{3}{32} \\ \frac{1}{64} & \frac{3}{32} & \frac{1}{64} \end{pmatrix} = \frac{1}{4}[\begin{pmatrix} \frac{1}{16} & \frac{3}{16} & 0 \\ \frac{3}{16} & \frac{9}{16} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & \frac{3}{16} & \frac{1}{16} \\ 0 & \frac{9}{16} & \frac{3}{16} \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ \frac{3}{16} & \frac{9}{16} & 0 \\ \frac{1}{16} & \frac{3}{16} & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{9}{16} & \frac{3}{16} \\ 0 & \frac{3}{16} & \frac{1}{16} \end{pmatrix}]$$

In the case of extraordinary vertex the mask of an even vertex for this scheme is shown in Figure(5), and the position of this vertex is given by:

$$v^{k+1} = \frac{9}{16}v^k + \frac{3}{8n}\sum_{i=1}^{n} e_i^k + \frac{1}{16n}\sum_{i=1}^{n} f_i^k \tag{7}$$

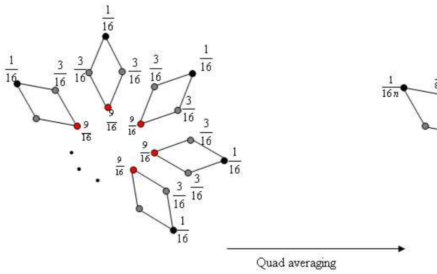

Figure 5: Quad-averaging mask

2.3 Boundary

When we encounter boundary vertices, we need to use boundary masks that are usually different from the interior mask. It is important that subdivision at any point on the boundary be independent of any point in the interior of the mesh. This permits two surfaces to be joined along a boundary curve. Therefore, cubic B-spline subdivision masks (Figure 6) for curves can be used as the boundary masks of Catmull-Clark subdivision.

• Cubic B-splines are a popular class of curves that are smooth and can be built with a simple subdivision scheme. Given a polygonal curve V^k , we denote the ith vertex of V^k by v_i^k. The edges of the polygonal curve are implicit in this representation since consecutive vertices (v_i^k and v_{i+1}^k) form an edge. The subdivision rules for cubic B-spline then have the form

$$v_{2i}^{k+1} = \frac{1}{8}v_{i-1}^k + \frac{3}{4}v_i^k + \frac{1}{8}v_{i+1}^k \tag{8}$$

$$v_{2i+1}^{k+1} = \frac{1}{2}v_i^k + \frac{1}{2}v_{i+1}^k \tag{9}$$

$$v_{2i-1}^{k+1} = \frac{1}{2}v_{i-1}^k + \frac{1}{2}v_i^k \tag{10}$$

Note that there are two rules, the number of vertices in the polygonal curve doubles during each round of subdivision.

In particular, the vertex v_i^k is repositioned to v_{2i}^{k+1} by the mask of even vertex $\{\frac{1}{8}, \frac{3}{4}, \frac{1}{8}\}$ while v_{2i+1}^{k+1} and v_{2i-1}^{k+1} are inserted at the midpoint of the edge v_i^k to v_{i+1}^k, and v_{i-1}^k to v_i^k by the mask of odd vertices $\{\frac{1}{2}, \frac{1}{2}\}$.

• The masks of even vertex and odd vertices of cubic B-splines form the subdivision mask for cubic B-splines $\{\frac{1}{8}, \frac{1}{2}, \frac{3}{4}, \frac{1}{2}, \frac{1}{8}\}$, the tensor product of this mask is the subdivision mask for Catmull-Clark in the regular case.

Let $S = \{\frac{1}{8}, \frac{1}{2}, \frac{3}{4}, \frac{1}{2}, \frac{1}{8}\}$ be the subdivision mask for cubic B-splines, this mask is given in term of generating functions by:

$$S(z) = \frac{1}{8}z^{-2} + \frac{1}{2}z^{-1} + \frac{3}{4} + \frac{1}{2}z + \frac{1}{8}z^2$$

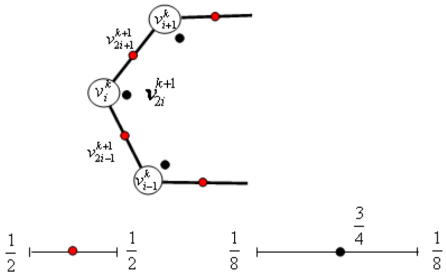

Figure 6: Cubic B-spline subdivision.

Then the bicubic B-spline surface can be expressed as the tensor product:

$S(z_1) \otimes S(z_2) = T(z_1, z_2) = Z_1 T Z_2^t$

where:

$\otimes$ is the tensor product,

$Z_1 = \begin{pmatrix} z_1^{-2} & z_1^{-1} & 1 & z_1 & z_1^2 \end{pmatrix}$,

$Z_2 = \begin{pmatrix} z_2^{-2} & z_2^{-1} & 1 & z_2 & z_2^2 \end{pmatrix}$ and

$$T = \frac{1}{64}\begin{pmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{pmatrix}$$

is the subdivision masks for Catmull-Clark in the regular case from which we get three different subdivision rules, one for the even vertex of the surface, one for the edge vertices and one for the face vertices (Figure 4).

It is easy to prove that the tensor product of the mask for even vertex of cubic B-splines is the Catmull-Clark mask of even vertex in the regular case:

$$\{\frac{1}{8}, \frac{3}{8}, \frac{1}{8}\} \otimes \{\frac{1}{8}, \frac{3}{8}, \frac{1}{8}\} = \begin{pmatrix} \frac{1}{64} & \frac{3}{32} & \frac{1}{64} \\ \frac{3}{32} & \frac{9}{16} & \frac{3}{32} \\ \frac{1}{64} & \frac{3}{32} & \frac{1}{64} \end{pmatrix}$$

We will use this property of subdivision to prove that the tensor product of the reverse mask for cubic B-splines is the reverse mask for a regular vertex of Catmull-Clark.

2.4 Reverse Mask for Catmull-Clark

In reverse subdivision, we know vertices of the level $k+1$ and we want to find the position of an even vertex v^k. In [1], Lanquetin and Neveu propose a reverse mask for the Catmull-Clark scheme for all arbitrary meshes (triangular, quadrilateral), which consists in locally reversing the originals formulas of edge vertices, face vertices and even vertex of Catmull-Clark(equation (1), (2) and (3)). The position of even vertex v^k is given by:

$$v^k = \frac{n}{n-3}v^{k+1} + \frac{4}{n(3-n)}\sum_{i=1}^{n} e_i^{k+1} + \frac{1}{n(n-3)}\sum_{1=1}^{n} f_i^{k+1} \tag{11}$$

More details can be found in[1]

But with this method we can not reverse other variants of quad schemes as Quad-averaging scheme of Warren and Weimer [2].

3. REVERSE SCHEMES FOR QUADRILATERAL MESHES

In this section we use an other method to determine the reverse mask of Catmull-Clark found in [1]. With this method our mask is parameterized and can also be used for reversing Quad-averaging scheme, and we will prove that we can use some properties of subdivision for the reverse subdivision, such as tensor product and the generalization of the vertex of Quad-averaging rule from the regular case to find the mask of arbitrary case:

• To apply the reverse Quad-averaging scheme to an arbitrary quad mesh, we need only to generalize the vertex rule from the valence four to vertices of arbitrary valence.

• The tensor product of the reverse mask of cubic B-splines is the reverse mask for a regular vertex of Catmull-Clark.

3.1 General Reverse Quadrilateral scheme

For the reverse process, it is necessary to construct a mask to map V^{k+1} to V^k. For an extraordinary vertex, assume the general situation shown in Figure 7. We know v^{k+1}, e_i^{k+1} ,f_i^{k+1} and we want to find v^k by a new mask such that the following conditions are met:

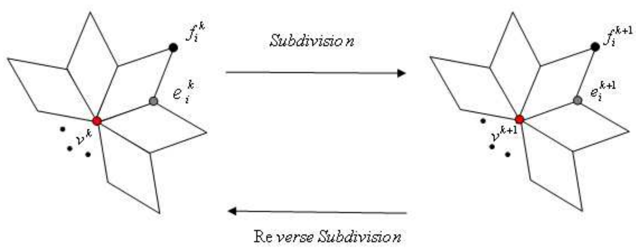

Figure 7: Situation for an extraordinary vertex

1• The operation of the new mask must be affine.

2•Weights of e_i^{k+1} in the mask must be equal. The same condition holds for weights of f_i^{k+1}. This is similar to the Catmull-Clark mask of equation (6)

3• The new mask must be a reverse of the subdivision mask i.e. the action of subdivision mask of Equation 4 , 5 and 6 on v^k and its neighbors must exactly reconstruct v^{k+1}. Condition (2) provides the diagram of Figure 8 for the reverse mask. In this diagram $\alpha^{'}$ is the weight of v^{k+1}, $\beta^{'}$ is the weight of e_i^{k+1} and $\gamma^{'}$ is the weight of f_i^{k+1} in the reverse mask. We now determine the weights $\alpha^{'}$, $\beta^{'}$ and $\gamma^{'}$ so that conditions (1) and (3) are also satisfied.

From condition (3) we have:

$v^k = \alpha^{'} v^{k+1} + \beta^{'} \sum_{i=1}^{n} e_i^{k+1} + \gamma^{'} \sum_{i=1}^{n} f_i^{k+1}$

From equations (4), (5) and (6), we obtain:

$$v^k = (\alpha\alpha^{'} + \tfrac{3}{8}\beta^{'} n + \tfrac{1}{4}\gamma^{'} n) v^k + (\alpha^{'}\beta + \tfrac{1}{2}\beta^{'} + \tfrac{1}{2}\gamma^{'}) \sum_{i=1}^{n} e_i^k$$

$$+(\alpha^{'}\gamma + \frac{1}{8}\beta^{'} + \frac{1}{4}\gamma^{'}) \sum_{i=1}^{n} f_i^k \tag{12}$$

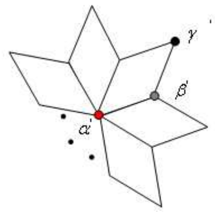

Figure 8: Situation for an extraordinary vertex.

From equation (12), we get the following system:

$$\begin{cases} \alpha\alpha' + \frac{3}{8}\beta' n + \frac{1}{4}\gamma' n = 1 \\ \alpha'\beta + \frac{1}{2}\beta' + \frac{1}{2}\gamma' = 0 \\ \alpha'\gamma + \frac{1}{8}\beta' + \frac{1}{4}\gamma' = 0 \end{cases} \tag{13}$$

We solve equation (13) with respect to α, β, γ. As $\alpha = 1 - n(\beta+\delta)$ we get a system with respect to β and γ:

$$\begin{cases} \alpha^{'} = \frac{1}{1-2n\beta} \\ \beta^{'} = \frac{4(2\gamma-\beta)}{1-2n\beta} \\ \gamma^{'} = -\frac{2(4\gamma-\beta)}{1-2n\beta} \end{cases} \tag{14}$$

with: $1 - 2n\beta \neq 0$.

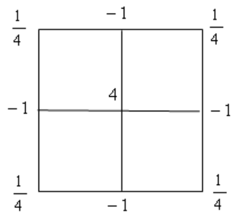

Figure 9: Reverse mask for regular vertex.

3.2 Reverse Catmull-Clark scheme

Equation (14) is a parametric formula for the reverse mask and can be applied for regular and extraordinary vertices.

In the case of regular vertex for all $(\alpha, \beta, \gamma) = (\frac{9}{16}, \frac{3}{32}, \frac{1}{64})$ we find:

$(\alpha', \beta', \gamma') = (4, -1, \frac{1}{4})$ (Figure 9).

For all $n \neq 3$ and for all ($\beta = \frac{3}{2n^2}, \gamma = \frac{1}{4n^2}$ we find:

$(\alpha', \beta', \gamma') = (\frac{n}{n-3}, \frac{4}{n(3-n)}, \frac{-1}{n(3-n)})$. Then the position of an even vertex is given by:

$v^k = \frac{n}{n-3} v^{k+1} + \frac{4}{n(3-n)} \sum_{i=1}^{n} e_i^{k+1} + \frac{1}{n(n-3)} \sum_{1=1}^{n} f_i^{k+1}$

We can see that with our method we get the same mask found in [1] equation (11) . In case of meshes with vertices of valence 3 more details can be found in [1].

3.3 Reverse mask for Quad averaging Scheme

Formula (14) is parameterized, and thus can be applied directly to Quad-averaging scheme. This implies that for all n we find:

$(\alpha', \beta', \gamma') = (4, \frac{-4}{n}, \frac{1}{n})$

We can see that in the regular case, the reverse mask can be decomposed into the sum of four sub-masks:

$$\begin{pmatrix} \frac{1}{4} & -1 & \frac{1}{4} \\ -1 & 4 & -1 \\ \frac{1}{4} & -1 & \frac{1}{4} \end{pmatrix} = \frac{1}{4}[\begin{pmatrix} 1 & -2 & 0 \\ -2 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -2 & 1 \\ 0 & 4 & -2 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & -2 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ -2 & 4 & 0 \\ 1 & -2 & 0 \end{pmatrix}]$$

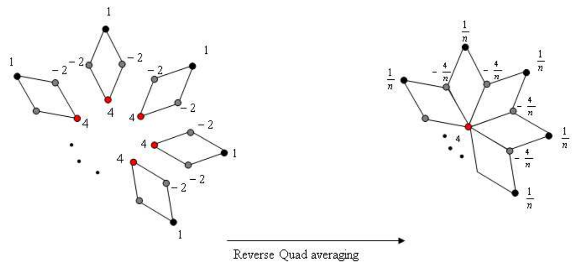

Figure 10: Reverse mask for Quad-averaging scheme .

To apply the reverse Quad-averaging mask to an arbitrary quad mesh, we only generalize the vertex rule from the valence four (Figure 9) to vertices of arbitrary valence illustrated in Figure 10.

3.4 Invariance Affine

In the case of regular vertex ($n = 4, \alpha = \frac{9}{16}, \beta = \frac{3}{32}, \gamma = \frac{1}{64}$), equation (14) gives $\alpha' = 4, \beta' = -1, \gamma' = \frac{1}{4}$ (Figure 9). In this case the sum of the weights is one. This property is generally correct for the Catmull-Clark and Quad-averaging schemes, since

$$\alpha' + n\beta' + n\gamma' = 1$$

this property holds for $n \neq 3$ in the case of Catmull-Clark, and for all n in the case of Quad-averaging.

3.5 Reverse Mask Of The Boundary Vertices

We have used cubic B-spline mask for boundary vertices. Therefore, we need to find a reverse mask for the cubic B-spline subdivision. In Bartels and Samavati, several masks for cubic B-spline subdivision are provided.

From equation (8), (9) and (10):

$$\begin{cases} v_{2i}^{k+1} = \frac{1}{8}v_{i-1}^k + \frac{3}{4}v_i^k + \frac{1}{8}v_{i+1}^k \\ v_{i-1}^k = 2v_{2i-1}^{k+1} - v_i^k \\ v_{i+1}^k = 2v_{2i+1}^{k+1} - v_i^k \end{cases}$$

Then:

$$v_i^k = -\frac{1}{2}v_{2i-1}^{k+1} + 2v_{2i}^{k+1} - \frac{1}{2}v_{2i+1}^{k+1}$$

This corresponds to the mask used in [13]

We can prove that as with subdivision the tensor product of the reverse mask of a cubic B-spline $\{\frac{-1}{2}, 2, \frac{-1}{2}\}$ is the reverse mask for a regular vertex of Catmull-Clark (Figure 9).

Let $M = \{-\frac{1}{2}, 2, -\frac{1}{2}\}$ be the reverse mask for cubic B-splines, we can write this mask in term of a generating fonction in the form:

$$M(z) = -\frac{1}{2}z^{-1} + 2 - \frac{1}{2}z$$

The tensor product:

$M(z_1) \otimes M(z_2) = M'(z_1, z_2) = Z_1 M' Z_2^t$

With:

$$Z_1 = (z_1^{-1}, 1, z_1)$$

$$Z_2 = (z_2^{-1}, 1, z_2)$$

$$M' = \begin{pmatrix} \frac{1}{4} & -1 & \frac{1}{4} \\ -1 & 4 & -1 \\ \frac{1}{4} & -1 & \frac{1}{4} \end{pmatrix}$$

We can see that M' is the reverse mask for Catmull-Clark in the regular case (see Figure 9).

4. RESULTS

To illustrate the method described in this paper, several examples are shown. The first example consists in rebuilding an initial mesh with vertices of valence 4 (torus) subdivided three times (Figure 11). The second example consists in rebuilding an initial mesh with extraordinary vertices subdivided by the Quad-averaging scheme (Figure 12). And the third example is a mesh with boundary subdivided by Quad-averaging scheme (Figure 13). In the three examples the successive meshes can be constructed with the general method described in the previous sections.

In the case of the meshes with vertices of valence 3 subdivided by Catmull-Clark scheme shown in [1]

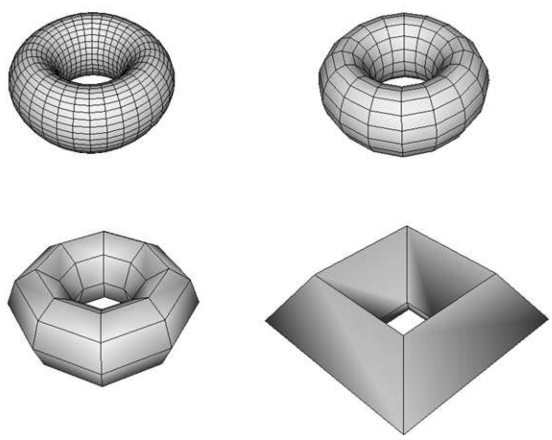

Figure 11: Reverse Catmull-Clark subdivision applied to on the torus mesh subdivided three times.

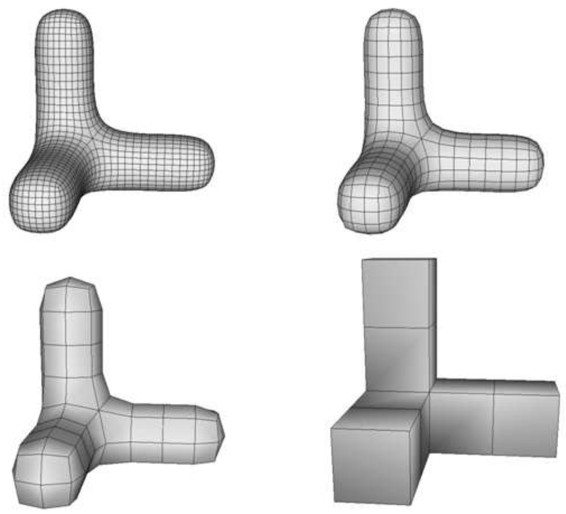

Figure 12: Reverse Quad-averaging scheme applied on a mesh with valences equal to 3, 4, 6 subdivided three times

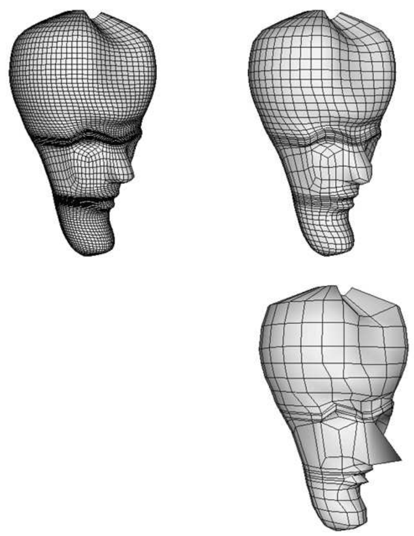

Figure 13: Reverse Quad-averaging Scheme applied on a mesh with boundary subdivided twice

5. CONCLUSION

We have used an other method to determine the reverse mask for Catmull-Clark found in[1]. With this method we have constructed a parameterized reverse mask for Catmull-Clark subdivision. The parameterization allows to reverse other quad schemes, such as the Quad-averaging scheme. we also have seen that we can use some properties of subdivision for the reverse subdivision, such as the tensor product and the quad averaging. This work proposes a general reverse scheme for quadrilateral structures that can be detected in irregular meshes using taubin's algorithms.

Future works will focus on mixed structures (i.e containing both triangular and quadrilateral faces). Tri-quad subdivision schemes have already been proposed. The problem is not solved for reverse tri-quad subdivision schemes.

ABOUT THE AUTHOR

Yacine Boumzaid is a PHD student at Burgundy University, Dijon , France, Laboratory of Electronics, Informatics and Image (LE2I). research on subdivision surfaces and reverse subdivision.contact email : Yacine.Boumzaid@u-bourgogne.fr.

Sandrine Lanquetin is an assistant professor at Burgundy University, Dijon, France, Laboratory of Electronics, Informatics and Image (LE2I). Research on subdivision surfaces. Contact email : sandrine.lanquetin@u-bourgogne.fr.

Marc Neveu is a full professor at Burgundy University , Dijon , France, Laboratory of Electronics, Informatics and Image (LE2I). His research concerns geometric modelling with a focus on deformable surfaces, subdivision surfaces and fractal surfaces. Contact email : marc.neveu@u-bourgogne.fr. Home page:http://danae.u-bourgogne.fr/Equipe/Pages/neveu/

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