Analysis of Unsteady Space-Time Structures Using the Optimization Problem Solution and Visualization Methods

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Abstract

This paper presents an approximate approach to analysis of space-time structures for unsteady problems in CFD (computational fluid dynamics). The approach is based on the solution of optimization problem combined with methods of data visual presentation. This approach is intended for fast approximate estimation of unsteady flow structures dependence on character parameters (or determining parameters) in a certain class of problems. For some cases such approach allows to obtain the sought-for dependence in a quasi-analytical form.

Ключевые слова: *unsteady problems, space-time structures, optimization, inverse problems, visualization methods*

1. INTRODUCTION

Time-dependent processes in CFD problems are often accompanied by the presence of changeable space-time structures (STS) in the flow, such as separation zones, circulating flows, vortex bursts, etc. These structures cause many undesirable effects in practice: reduced lift, airframe and control vibrations. STS can appear and disappear defining the flow pattern and quantitative characteristics of the flow field. Simulating these changeable structures is therefore an important aspect of CFD.

Nowadays, modern computer hardware and mathematical methods allow one to simulate practically any time-dependent physical process in CFD and to obtain corresponding field of physical values. Calculating thoroughly the flow field one can obtain a beautiful picture of STS transformations. Nevertheless it is evidently insufficient for practical aims. In practice the phenomenon (physical effect) by itself is not the main point of interest. For practical engineering it is more interesting to know the circumstances leading to the phenomenon appearance, i.e. how this appearance depends on the problem character parameters, such as Mach number, Reynolds number, Prandtl number etc. To define such dependence one should solve the problem in optimization statement, which is based on multiple calculations of the inverse problem.

This paper presents an approximate approach to analysis of space-time structures. This approach is intended for fast and rough estimation of unsteady flow structures dependence on character parameters in a certain class of problems. The approach is based on the solution of optimization problem combined with methods of data visual presentation. Visualization methods are applied to the solution of optimization problem. The solution is obtained in a form of multidimensional array. According to classification described in [1], such approach can be referred to *data analysis methods.*

According to [2], there are two high-priority tasks for parallel computations: multidisciplinary problems and inverse problems. From this point of view the described below approach is very promising because it can be applied to a very wide range of time-dependent processes for various practical applications. Using the theory of optimization problems solution the approach is a modification of method [3] (parametric space analysis).

As it is shown below for concrete examples, this approach allows obtaining the sought-for dependencies in a form of quasi-analytical expressions.

2. INVERSE PROBLEMS AND CFD APPLICATIONS

Numerical computation of the inverse problems in CFD is enough difficult. One should calculate 4D problems (3D+time) in a *variational statement*. It requires using high-performance computers. The separate difficulty is to visualize the inverse problems solutions for multidimensional case. There is a significant lack of concepts and tools in scientific visualization now. Nevertheless, the rapid development of computing technologies and hardware allow us to solve this class of numerical simulation problems.

We can solve a wide range of CFD problems using the concept and statement of inverse problems. Typically, the practical engineering problem is to choose the desired variants from the set of admissible variants. This can be the choice of a geometric shape (minimal drag), the choice of flow control (maximal mixing), etc. According to [4,5], the inverse problems are classified as boundary searches, coefficient searches, retrospective inverse problems, optimization problems. In general, for practical goals inverse problems are formulated as follows. One should find the determining parameters for which a phenomenon of interest occurs in a certain class of problems. It shouldn't depend on the details of the phenomenon appearance. It can be quantitative appearance of the phenomenon (some variable reaches definite value) or qualitative appearance (vortex formation, flow separation, any other STS changing).

Let's consider formalized statement of the inverse problem in a general form.

The numerical solution of the chosen CFD problem is elaborated during the computation process. The solution is defined by the finite set of determining parameters (or character parameters) of the problem. These determining parameters can be divided into three main groups: $\boldsymbol{A} = (\boldsymbol{a}_1,...,\boldsymbol{a}_n)$ - the parameters determining physical properties and mathematical model of problem; $\boldsymbol{B} = (\boldsymbol{b}_1,...,\boldsymbol{b}_m)$ - the parameters determining the numerical method; $\boldsymbol{C} = (\boldsymbol{c}_1,...,\boldsymbol{c}_l)$ - the parameters determining organization of the calculation process.

All these parameters form the numerical solution $F = F(A,B,C) = F(a_1,...,a_n,b_1,...,b_m,c_1,...,c_l)$ as a result of computation. So, the solution is based on the chosen mathematical model, numerical method and the way of calculation organization.

We consider *the event functional* $\Phi(F(A,B,C))$. Just as logical variable so this functional has two values: $\Phi(F(A,B,C)) = 1$ – if the event of interest occurs (independently on the event details), or $\Phi(F(A,B,C)) = 0$ - if the event of interest doesn't occur.

Presenting $\Phi(F(A,B,C))$ in a form $\Phi(F(A,B,C)) = \Phi(a_1,...,a_n,b_1,...,b_m,c_1,...,c_l)$ one can formulate the inverse problem in a general form as follows. Find all the determining parameters $(a_1,...,a_n,b_1,...,b_m,c_1,...,c_l)$ for which a phenomenon of interest occurs in a certain class of problems, i.e. $\Phi(a_1,...,a_n,b_1,...,b_m,c_1,...,c_l) = 1$ (2.1)

Considering the determining parameters $(a_1,...,a_n,b_1,...,b_m,c_1,...,c_l)$ as a set of basis vectors, one can present the space of the determining parameters $L(a_1,...,a_n,b_1,...,b_m,c_1,...,c_l)$ having $(n+m+l)$ dimensions.

Then for general case the inverse problem can be formulated as the problem of finding in this space L all the subdomains L^* where the event of interest is observed, i.e. $\Phi(L^*) = 1$.

At the same time the problem of data filtration is solved. Setting the ranges for character parameters one can not guarantee the fact of appearance of the sought-for event inside the range. So if the event does not occur for some point of space, this point is not considered.

3. OPTIMIZATION PROBLEM AND VISUALIZATION

Using numerical or experimental modeling of unsteady phenomenon for practical goals in mechanics we usually know the reason of phenomenon appearance and quantitative parameter regulating this reason (control parameter) f_{cont}^*. The simulation is intended to define the control parameter dependencies on the determining parameters $(f_1,..., f_n)$ of the problem. To obtain such dependencies $f_{cont}^*(f_1,..., f_n)$ in a quasi-analytical or in a table form is a real practical goal of research. As a matter of fact these dependencies have been the main point of practical CFD applications last 50 years.

This paper considers a methodological approach to obtain these dependencies by means of numerical simulation. The approach can be described in general as follows.

Let's suppose that one has mathematical model of the CFD problem and reliable numerical method for solving. Then one can compute the straight problem of unsteady process simulation. During this simulation some event occurs.

To study thoroughly the unsteady event one should solve the inverse problem with purpose to find the exact value of control parameter, when the event (STS transformation, for instance) occurs.

To solve inverse problem one should multiply solve the straight problem varying the control parameter $f_{cont}^*(f_1,..., f_n)$ until the onset of the unsteady phenomenon (physical effect) of interest. During this optimization process the set of determining parameters $(f_1,..., f_n)$ is fixed.

Then the determining parameters $(f_1,..., f_n)$ are varied with chosen variation step and the inverse problem is repeatedly solved for each set of determining parameters. As a result of such computations we obtain control parameter dependence on determining parameters in general form of n-dimensional array $f_{cont}^*(f_1,..., f_n)$.

This form is not suitable for practical goals. The most effective way of a search of the sought-for dependence in a quasi-analytical form is visual presentation of the array.

Analyzing the array one can decrease the dimensionality. For this purpose one should omit those determining parameters, which do not influence at the control parameter. If one has as a result $n \le 3$ after such decreasing, then the rest of data can be visualized. For some cases the visualization is fast and effective way to obtain the dependence $f_{cont}^*(f_1,..., f_n)$ in a form of quasi-analytical expression. To obtain such expressions it is assumed to approximate the array data (where it is possible) by simple geometric elements, such as lines, planes, parts of spheres etc.

It is very simple to do if we are dealing with the case of two determining parameters. We can approximate the surface $f_{cont}^*(f_1, f_2)$. For the case of three determining parameters we are able to build the isosurfaces. Then we can try approximating the isosurfaces by means of simple geometric elements. But for the case $n > 3$ there is an evident lack of concepts and tools for visual presentation. The creation of reliable and suitable for human acceptance visual presentations for this case is a subject for discussions now.

By tradition the problem of multidimensional data visualization can be referred to the field of *Information Visualization* due to the fact that the solutions of such problems are necessary for business applications. There were some attempts to elaborate original visualization methods for multidimensional data. For instance, one can mention such approach as "Chernov's faces" [6]. The main idea of this approach is to present the values of different variables by various details of human face. Another attempt is presented in [7] for the space of events as "event tunnel'. The space of events is presented in a form of 3D cylinder ("event tunnel'). Lengthwise axis presents the time; the events are presented as spheres inside the cylinder. As the distance from the point of observation grows, the sizes of spheres are reduced. Despite some success for business applications the artificial character of such visual concepts is evident.

So for common case one should try to decrease the array dimensionality up to 3 and hope the class of problems under consideration would allow such decreasing. Fortunately, for many real applications it is true, as it is shown below.

There are some well-known ways to decrease the array dimensionality. One of these ways is analysis of variances for each character parameter. Character parameter is considered as coordinate direction. The direction with minimal variances is rejected (compactification). Another way is the construction of different 3D projections for various triplets of determining parameters, as it is shown below for example of computations. Also one can apply PCA method (Principal Component Analysis)

[8]. This method is based on localization of 3 principal components and data presentation using these components as new coordinate system. Combining these methods one can decrease the array dimensionality for many practical cases.

Our approach is based on optimization. Therefore, it amounts to solving a set of similar small tasks. Each small task is the solution of inverse problem having a fixed set of determining parameters. So the approach corresponds to task parallelism ideology. Using the principle “one task – one processor” and having a minimal quantity of internal exchanges allows one to make very effective applications of this approach to practical problems. The possibility of rough grids using is another one serious advantage of the approach to be described.

The approach can be applied to analysis of finite difference schemes also. This methodological approach was successfully used in [9] to optimize the computational properties of hybrid finite difference scheme applied to the solution of the supersonic far wake problem. Viscosity and turbulence were taken into account. The emergence of undesired oscillations was considered as the event to be controlled. The weight coefficient of finite difference scheme was used as the control parameter. The grid step, Mach and Reynolds numbers were chosen as determining parameters. As a result for chosen class of problems the weight parameter dependence on determining parameters was obtained in a quasi-analytical form.

4. APPLICATION EXAMPLES

The optimization approach is applied to analysis of unsteady circulating zones transformation. The problem of unsteady interaction of the supersonic viscous flow with jet obstacle is considered. This obstacle appears due to co-current underexpanded jet exhausting from the nozzle. The nozzle is placed to external supersonic viscous flow. Expanding jet propagates on the external surface of the nozzle and creates obstacle in external flowfield. The obstacle disturbs external flow and circulating zone appears ahead the obstacle. Typical flow structure is shown in Fig.1 by streamlines.

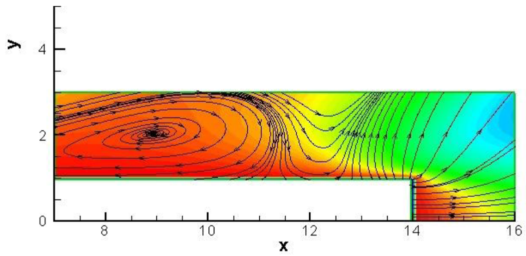

Figure 1: Flow structure for slow pressure ratio growth.

We consider a problem containing time-dependent boundary condition for underexpanded jet. Jet pressure ratio was set at the nozzle edge as time-dependent function $\boldsymbol{n}=\boldsymbol{n}(\boldsymbol{t})=\boldsymbol{P_a}/\boldsymbol{P_\infty}$ (where $\boldsymbol{P_a}$ - jet pressure, $\boldsymbol{P_\infty}$ - external flow pressure). The full system of time-dependent Navier-Stokes equations for viscous compressible heat-conductive flow is used as mathematical model. Implicit hybrid finite difference WW-scheme is applied to solve the system of equations. This scheme has second order accuracy in time and space.

The dependence $\boldsymbol{n}=\boldsymbol{n}(\boldsymbol{t})$ is chosen as linear function. It allows one to set different rates of pressure ratio growth up to $\boldsymbol{n}=\mathbf{100}$.

As a result of calculations of straight problem a new STS formation is obtained. Increasing the rate of pressure ratio growth one obtains new space-time structure in the vicinity of circulating zone ahead the jet obstacle. This new structure is shown by streamlines in Fig.2.

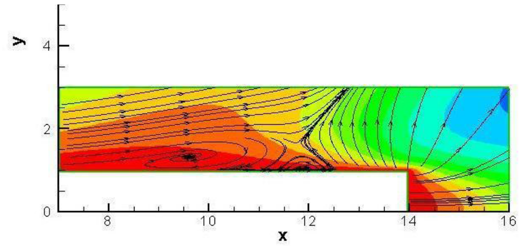

Figure 2: Flow structure for fast pressure ratio growth.

Let’s consider the optimization approach application to analysis of unsteady event: the formation of the new space-time structure in the flow. The rate of pressure ratio growth is chosen as control parameter. The case of four determining parameters is considered. These four parameters are Mach number $\boldsymbol{M_\infty}$, Reynolds number $\mathbf{Re_\infty}$, Prandtl number - $\mathbf{Pr_\infty}$ and $\boldsymbol{Sh_\infty}$ - Strouhal number for the problem under consideration. For each fixed set of these numbers $(\boldsymbol{M_\infty},\mathbf{Re_\infty},\mathbf{Pr_\infty},\boldsymbol{Sh_\infty})$ the inverse problem is solved by varying pressure ratio growth rate until the onset of the new structure formation in the flow. These character numbers vary in ranges: $\mathbf{1.5}\le\boldsymbol{M_\infty}\le\mathbf{3}$; $\mathbf{2.5}\le\mathbf{lg\,Re_\infty}\le\mathbf{4}$; $\mathbf{0.72}\le\mathbf{Pr_\infty}\le\mathbf{1}$; $\mathbf{1}\le\boldsymbol{Sh_\infty}\le\mathbf{2}$. For each new fixed set of numbers $(\boldsymbol{M_\infty},\mathbf{Re_\infty},\mathbf{Pr_\infty},\boldsymbol{Sh_\infty})$ the procedure described above is repeated.

So for each set of determining parameters $(\boldsymbol{M_\infty},\mathbf{Re_\infty},\mathbf{Pr_\infty},\boldsymbol{Sh_\infty})$ one defines the exact value of pressure ratio growth rate when the new flow structure appears. Using this value one can just define the character time $\boldsymbol{t^*}$ and crucial velocity of pressure ratio growth $\boldsymbol{V^*}$ for the new STS appearance. As event character time $\boldsymbol{t^*}=\boldsymbol{t_{ev}}/\boldsymbol{t_{n=100}}$ is chosen, here $\boldsymbol{t_{ev}}$ - the time when the event occurs and $\boldsymbol{t_{n=100}}$ - the time when pressure ratio reaches 100.

For computations two algorithms are elaborated – serial and parallel. Two types of grids are chosen: 5 and 10 points for each determining parameter. It requires computing 625 and 10000 inverse problems. The computations are performed by parallel complex K100. MPI technology is applied to control parallel computations.

As a result of approach application four-dimensional arrays are obtained. These arrays contain numerical presentations of the character time $\boldsymbol{t^*}$ and crucial velocity $\boldsymbol{V^*}$ dependencies on four determining parameters $(\boldsymbol{M_\infty},\mathbf{Re_\infty},\mathbf{Pr_\infty},\boldsymbol{Sh_\infty})$. The dependencies $\boldsymbol{V^*}=\boldsymbol{V^*}(\boldsymbol{M_\infty},\mathbf{lgRe_\infty},\mathbf{Pr_\infty})$ and $\boldsymbol{t^*}=\boldsymbol{t^*}(\boldsymbol{M_\infty},\mathbf{lgRe_\infty},\mathbf{Pr_\infty})$ are presented in Fig.3,4 by isosurfaces. The view of these isosurfaces shows that character time and crucial velocity does not depend on Reynolds number for chosen class of problems in this laminar diapason of $\mathbf{Re}_\infty$.

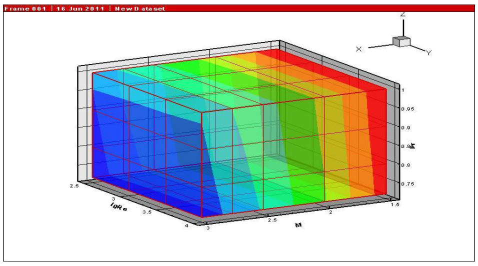

Figure 3: Crucial velocity dependence on Mach, Reynolds and Prandtl numbers

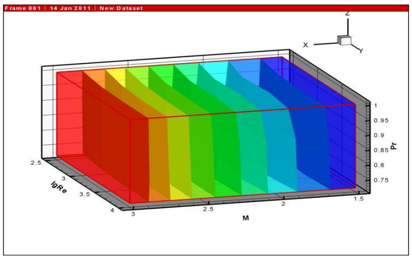

Figure 4: Character time dependence on Mach, Reynolds and Prandtl numbers

Further, the dimensionality of array can be decreased and one is able to consider 3D arrays $V^* = V^*(M_\infty, \mathrm{Pr}_\infty, Sh_\infty)$ and $t^* = t^*(M_\infty, \mathrm{Pr}_\infty, Sh_\infty)$. These dependencies are shown in Fig.5,6 by isosurfaces.

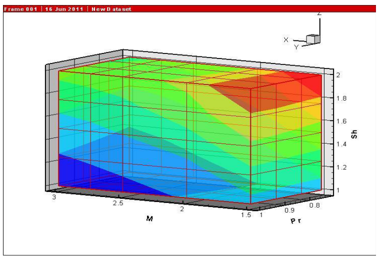

Figure 5: Crucial velocity dependence on Mach, Prandtl and Strouhal numbers

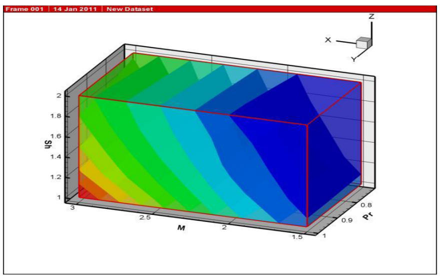

Figure 6: Character time dependence on Mach, Prandtl and Strouhal numbers

Analyzing the view of these isosurfaces one can approximate the isosurfaces by planes. For the purpose of rough estimation the sought–for dependence can be written in a form of plane

$$AM_\infty + B\,\mathrm{Pr}_\infty + CSh_\infty = const\ .$$

It allows one to get average estimation of V^* and t^* dependencies on determining parameters as

$$V^* = V^*(M_\infty, \mathrm{Pr}_\infty, Sh_\infty) = -0.1M_\infty + 0.115\,\mathrm{Pr}_\infty + 0.24Sh_\infty$$

$$t^* = t^*(M_\infty, \mathrm{Pr}_\infty, Sh_\infty) = 0.224\,M_\infty - 0.04\,\mathrm{Pr}_\infty - 0.132Sh_\infty$$

These expressions describe the connection between control parameter (pressure ratio growth rate) and character parameters – Mach, Reynolds and Strouhal numbers.

The way to check the accuracy of such approach is simple enough. For this purpose one should specify character numbers and the rate of pressure ratio growth and then one should carry out the computation of the straight problem. The resulting space-time structure can be compared with the structure corresponding to approximate expression.

5. CONCLUSIONS

The optimization approach to unsteady processes analysis is considered. The approach allows carrying out fast and effective estimation of the way how the crucial points of flow structure transformation depend on determining parameters of the problem. Combining the inverse problems solutions with the visual presentations of such solutions for many real cases allows one to obtain the sought-for dependencies in a quasi-analytical form. The approach can be applied for rough grids. The approach amounts to solving a set of similar small tasks, so it can be applied also for parallel computations. The approach can be applied to a wide range of time-dependent processes for various practical applications.

6. REFERENCES

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