Rendering of transparent optically anisotropic objects

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Abstract

At the present time, computer graphics demonstrates a lot of achievements in physically correct rendering of opaque objects and optically isotropic transparent and translucent objects (such as glass, diamond, or water).

This paper examines the current state of art in the algorithms for physically accurate calculation of images of 3D scenes including transparent optically anisotropic objects (feldspar, KDP, etc.). It should be noted that very little work has been made in this field, with the most important of it in this century. Only rendering of transparent optically anisotropic objects (without absorption) is considered here.

Keywords: *photorealistic rendering, transparent objects, optical dispersion, optical anisotropy, birefringence, polarized light, crystals.*

1. INTRODUCTION

There exist several major algorithms for rendering of 3D scenes [1, 8, 39]: scanline algorithms like OpenGL and DirectX, ray casting, Whitted style ray tracing (WSRT), radiosity, and Monte Carlo ray tracing. Specific rendering algorithms are their combinations and modifications. It is well-known that all algorithms require some specifications of the materials of scene objects. The material of an object determines how light interacts with this object. A description of the parameters of light coming to a point of a scene or a scene object, that is, the directions of reflection and the fraction of light energy reflected, is called a Local Model of Light Interaction (LMLI) with an object. The recent developments of computer graphics and increase in computer power make it possible to use more complex materials and more sophisticated LMLIs to correctly characterize this interaction from a physical point of view.

Major attention will be given to crystalline scene objects: monocrystals and crystalline aggregates, although the algorithms being considered can be applied to any transparent medium with a piecewise-smooth boundary. It should be noted that the behavior of many natural objects that are normally optically isotropic can be anisotropic under certain physical conditions (external fields, etc.), that is, they demonstrate birefringence, for instance, even vacuum [35]. Therefore, an intention to more and more correctly render the various scenes may require to use the rendering algorithms considered here even for typical materials, such as glass. It is appropriate to recall the following Kajiya's statement [12]: "A thread that runs throughout computer graphics is the quest for detail. Nature presents a nearly infinite complexity and richness of form over an enormous range of scales. In image synthesis it is our task to make convincing pictures of such natural phenomena: thus how to represent this range of scales becomes a central problem. "

The present paper is a condensed version of our talk. It is organized as follows: Section 2 provides a scheme of rendering based on ray tracing. Section 3 gives a brief insight into the evolution of the materials. Section 4 deals with a Local Model of Light Interaction with crystals, and Section 5 presents developments of algorithms to render transparent optically isotropic and anisotropic objects. Section 6 is devoted to testing/verification. Section 7 discusses some rendering problems closely related to those described above.

2. RENDERING

Consider the problem of photorealistic rendering in the following statement.

A scene is a set of objects, surfaces, and light sources. They are located in the space of the scene. An object is given in some way, and has a closed boundary. Each boundary divides two media with specific optical characteristics. We calculate the scene image for a given location of the camera with the following input data:

- Procedures for finding the point of intersection of the ray with the object boundaries (or scene surfaces).
- At the boundary point, there exists a normal, and it can be calculated.
- At the boundary point, the materials of both media are specified.
- An LMLI is specified for each type of boundary between different objects/media.
- A material is specified at each point of every particular medium.
- A material is specified for the medium filling the scene space. The most-used approach is to assume that the scene medium is vacuum.

The process of rendering is based on the construction of light ray paths: from a source or camera to an object, from one object to another, or inside transparent or translucent objects. The most important characteristic of rays of the paths is the energy transferred by them. The resulting photorealistic image is obtained by calculating this energy.

3. TRADITIONAL MATERIALS

3.1 Bidirectional reflectance distribution function

A popular local model for characterizing the material of points of opaque surfaces is the bidirectional reflectance distribution function $BRDF(\omega_i, \omega_r)$, which shows the portion of energy coming from direction ω_i and reflected in direction ω_r. This is *isotropic reflection,* since it depends only on these angles. It has the property of Helmholtz reciprocity: $BRDF(\omega_i, \omega_r) = BRDF(\omega_r, \omega_i)$. The existing BRDFs can be roughly subdivided into those obtained from simulation and from geometrical and wave optics. In most photorealistic rendering algorithms, it is sufficient to specify the material only at the points of boundaries between the objects/media. There are more general BRDFs, for instance, anisotropic reflection ones [12], where, in addition to the angles of incidence and reflection, some special directions associated with the surface itself are taken into account.

The phenomenological BRDFs can be subdivided into:

- *Intuitive BRDFs*. Example: Phong's specular reflection formula [39]. It is clear intuitively (according to Phong) that $\cos^{shineness}(\alpha)$ makes it possible to rather realistically display highlights in some applications.
- *Simulation BRDFs*, obtained in natural experiments with the material under study in a special setup that makes it possible to estimate even anisotropic reflection [38].

Modeling BRDFs are obtained, as a rule, on the basis of optical laws.

Within the class of opaque objects, some successful attempts have been made to create BRDFs for objects with subsurface light scattering (skin, marble, and even leaves) [9, 11].

For transparent and translucent objects, the material is described, in addition to the BRDF, also by the bidirectional transmission distribution function (BTDF), and the total behavior of light is represented by the bidirectional scattering distribution function (BSDF) at a surface point [22].

3.2 Spectral rendering

The recent increase in computer efficiency caused the practical usage of spectral rendering (for instance, MaxwellRender – http://www.maxwellrender.com/). In fact, a phenomenon like light dispersion cannot be simulated within the framework of a color model RGB to illustrate Newton's experiment on white light splitting, well-known from school physics. Here, more complicated specifications of materials in the form of spectra – $BSDF(\lambda, \omega_i, \omega_r), \lambda = 380..780nm$ are used, that is, the light scattering function is specified for some set of waves from the visible spectrum. The spectra of light sources are also given. For this more precise model, the following problems are solved:

- Transformation of RGB images into spectral representation. This is a practical problem, since most initial data are given in the RGB format.
- Transformation of the spectral representation of an image into the RGB format (*tone reproduction*). The obtained result image in spectral form should be displayed using available RGB-devices with a poorer color range to obtain images producing the same impressions of the observer. Numerous approaches have been proposed to solve this problem (see [6]).

Obviously that researchers focused their attention on the rendering of glass products and diamonds [6, 18, 19] taking into account absorption in the object medium [20]. The laws of optics (the Buger-Lambert-Baer law) were used to derive a model of absorption [28].

3.3 Polarized light

Nevertheless, a lot of objects still have to be studied, since no physically correct models are available in computer graphics for the interaction of light with translucent objects and media, as well as with metals. This is due to the fact that the models in use had some assumptions, which were characterized by Hanrahan in the introduction chapter 2 to book [1] (1993) as follows: "The polarization of incident radiation is an important parameter affecting the reflection of light from a surface, but the discussion will be simplified by ignoring polarization". Up to now, only unpolarized light has been considered in commercial programs of rendering. As a rule, nothing is said about the polarization of light. This is a justified approach, since in typical scenes the light (except for solar light) is often weakly polarized [6].

The fact that some people can distinguish polarized light from unpolarized light cannot be ignored [17].

Light polarization in optics is a very important concept. However, of interest to us are only papers on photorealistic rendering. Using various kinds of light polarization, one can obtain a variety of images. Consider a graphic example demonstrating the concept of Brewster's angle (the angle of incidence at which full polarization of reflected light takes place) [27, 28] where the light reflected from a water surface is fully polarized. In this way, *full refraction can be achieved* (http://ru.wikipedia.org/wiki/Закон_Брюстера).

4. LMLI OF CRYSTALS

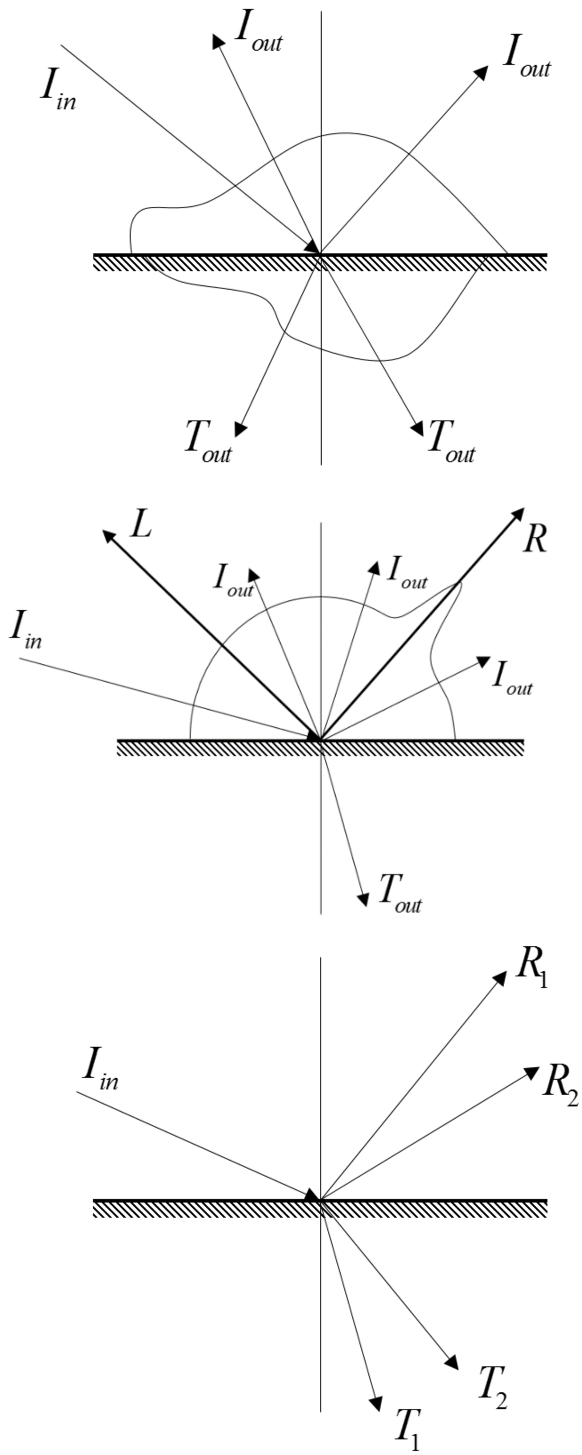

Figure 1: Different models of light ray-surface interaction.

Fig. 1 (top) presents a typical BSDF, which is the most general case of behavior of a ray (or path) after it falls on the surface of a translucent object. The possible directions of reflection I_{out} and refraction T_{out} and the portions of energy going in these directions are taken into account. A function that is typical for the Whitted algorithm of ray tracing is shown in the center of Fig. 1 [25, 39]. Here L is the direction to the point light, R is the corresponding vector of the reflected ray, and T_{out} is the vector of the refracted ray. Whereas for the incoming ray I_{in} the reflection distribution function is fully represented by the outgoing rays I_{out} in the model, refraction takes place only along the single ray T_{out}, which depends on I_{in} and is calculated by the Snell law. Fig. 1 (bottom) shows the most general case of response of a transparent crystal surface to an incoming ray I_{in}. Reflection in one or two directions of reflection (R_1 and R_2) and refraction in one or two directions of refraction (T_1 and T_2) are possible. Hence, the incident ray can give a maximum of only four directions of outgoing rays.

Thus, the BSDFs are piecewise-continuous functions of directions. This key property of the functions is a basis of many rendering algorithms. The continuity allows one to perform linear approximations, specify these functions with the help of tables, etc. As for crystals, this is the sum of delta-functions.

As for ray tracing in scene rendering, a ray must contain, in addition to geometrical information, some information about the polarization of light carried by this ray. Several concepts of polarization have been successfully used in solving the problem of rendering: coherence matrices, Stokes vectors, and Mueller matrices (see, for instance, [6, 28]).

From an optical point of view, all transparent and translucent media can be subdivided into isotropic and anisotropic ones. In isotropic (semi)transparent media (glass, diamond, water) all directions of light propagation are equal, whereas the anisotropic media effects on light propagating in different directions will differ from one another. Well-known representatives of anisotropic media are crystals, for instance, feldspar (calcite). Anisotropic media have specific directions, called *optical axes*. *Isotropic* media do not have optical axes. If a medium has only one axis, it is called *uniaxial*, and if it has two axes, it is called *biaxial*. An important property of optically anisotropic media is birefringence (that is, double imaging) and double reflection (see Fig. 1, bottom). Thus, one or two reflected and one or two refracted rays can be generated at the boundary of anisotropic media for a single incident ray.

Fully transparent media are important, because absorption (translucence, medium's color) and optical activity [28, 15, 16] are significant only inside the medium. However, the direction of (reflected and refracted) rays generated by an incident ray is calculated in the same way both in absorbing and transparent media.

The first important works that appeared in the last century are as follows:

- [27] – a mathematical model of the phenomenon of polarization which can be applied in computer graphics is constructed. This model is based on coherence matrices and takes into account a possibility of partial ray polarization, elliptic polarization. A possibility of polarization under reflection from surfaces of dielectrics with incidence under Brewster's angle is also considered. However, in this case only reflection from isotropic dielectrics is taken into account. The question of the degree of polarization of a ray reflected from the surface of a birefringent crystal still remains open.
- [14] – the problem of ray incidence from an isotropic medium to a uniaxial one is considered. Formulas to calculate the directions of generated rays are presented.
- [15, 16] – these papers are devoted to ray tracing. The directions and degrees of polarization of generated (reflected and refracted) rays are calculated. Thus, a calculation of energy is performed, which makes it possible to correctly calculate the intensity of light energy. Uniaxial absorbing and active crystals are considered.
- [21] – an algorithm to simulate birefringence in uniaxial crystals, specifically, simulation of polarization effects using coherence matrices, is considered. **This is the first paper** which presents an image for a test scene including calcite and a polarizing filter; the *test image is colored*.

The following papers of interest appeared in the 21st century:

- In paper [32], the first attempt is made to formulate the problem of crystal rendering.
- Determination of generated rays (without calculation of polarization) is made in [13, 14, 33, 34, 40, 41].
- Determination of generated rays (with calculation of polarization) is made in [5, 10, 23, 24, 30, 36]. Paper [5] is an extension of [30, 36]. Paper [23] describes a theory, without any experiments.
- A test image is presented and calculated in [5, 10, 24, 30].

Papers [14–16, 23] were used in the development of LMLIs for crystals.

There are a lot of important papers that should be included in this context, but they are typically devoted to finer optical effects and are not mentioned in papers on computer graphics of anisotropic objects.

The calculation formulas are based on the physical laws of electromagnetic wave propagation by solving Maxwell's equations. The fundamental references are books [28, 42].

The derivation of major formulas for the calculation of generated rays is very important. Since the books on crystal optics usually do not care about applications of computer graphics and, therefore, do not contain the required formulas, the formulas are derived by specialists in photorealistic rendering. A geometrical / trigonometrical approach is mainly used, although in some papers matrix calculations are used [15, 16]. A covariant method [42] is used in [5, 30]. It should be noted that many of the papers do not consider the purely isotropic case at all.

The domain of applicability of a local model of light interaction is defined by the available formulas. Each object or medium in a scene can be characterized from an optical point of view in the following way: 0 – isotropic medium, 1 – uniaxial medium, 2 – biaxial medium. Then the types of boundary between objects and/or media can be denoted as $i \Rightarrow j$, that is, the LMLI calculates the generated rays if a ray falls from a medium i to the boundary with a medium j. All above-mentioned papers are devoted to LMLIs providing formulas for boundaries with one isotropic medium. Only paper [5] proposes a unified LMLI algorithm for any boundary type. In nature, minerals most often have the form of crystalline aggregates (“conjoined monocrystalline parts”). Thus, a LMLI for the boundary between two anisotropic media is necessary.

Also, one should mention an attempt to formalize a material in the form of a VRML [7] extension made in paper [30].

5. RENDERING OF CRYSTALS

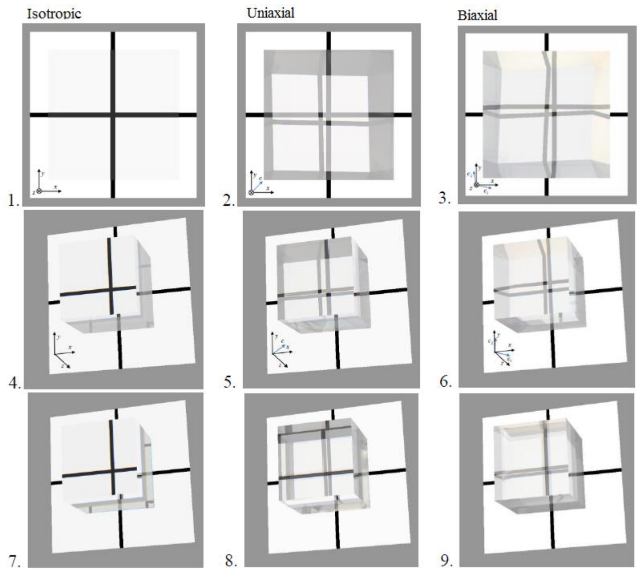

Figure 2: Variants of a virtual scene.

Real rendering of test scenes with crystals is presented in papers [4, 10, 13, 30]. Fig. 2 presents nine variants of a virtual scene consisting of a transparent cube. The cube is lying on a plane white square with a black cross drawn on it. The left column (cells 1, 4, 7) has cubes from isotropic materials, the middle column (cells 2, 5, 8) has cubes from uniaxial materials, and the right one (cells 3, 6, 9), cubes from biaxial materials. In the two upper rows of images (cells 1 – 6) the cube is made from a monocrystal, only for the top row the camera is located right in the center, and in the second row the camera looks from one side. In the lower row (cells 7 – 9) the cubes are aggregates. They consist of two layers - two half-cubes with different optical characteristics: cell 7 – layers with different refraction coefficients, cells 8 and 9 – layers with different directions of optical axes.

- The images in the left-column cells can be obtained using numerous commercial programs (for instance, Maxwell).
- Using algorithms [10, 24], one can obtain images in cells 2 and 5.
- Using algorithm [13], one can obtain "sketches" for images in cells 2, 3, 5, and 6. In this case the colors will not be correct, because energy balance cannot be calculated.
- Using algorithm [5], one can obtain any of the nine images.

Some more remarks should be made to provide a better understanding of the peculiarities of rendering scenes with transparent objects.

In the process of rendering, the following light ray paths are constructed: from a source to an object, from one object to another, inside transparent and translucent objects.

A polarized ray is considered in [5] as a structure containing: λ – light wavelength; P_0 – ray origin point; $\vec{r}_d$ – direction vector (ray vector); $\vec{m}$ – phase propagation vector; CS – system of coordinates of the ray; J – 2x2 coherence matrix; ray type (isotropic, ordinary, extraordinary, fast, slow [28]). CS and J define the degree of ray polarization. All these parameters are calculated after determination of the directions of the generated rays.

With recursive algorithms of backwards tracing like the Whitted algorithm [25], a tree of backwards tracing is constructed. At each node, up to 4 descendants (Fig. 1, bottom), and not 2, as in the case of isotropic objects (Fig. 1, center), are generated. Only paper [31] describes an implementation of tracing and branching into 4 descendants. In the other papers, no attention is given to this question. Once the tree of paths from the camera is constructed, it is passed in the reverse order to collect the energy.

It is much more expensive to calculate the polarization of generated rays rather than the direction of the generated rays themselves. Therefore, polarization is not calculated in the tree construction. In the process of energy collection from the leaves (light sources) to the root, for every ray connecting a descendant node with its predecessor all its generated rays (as for an incident ray) and the degree of their polarization are fully calculated.

It should be noted that under reflection from a diffuse surface the ray becomes unpolarized, regardless of the state of polarization of the incident ray [27].

6. VERIFICATION

Traditionally, expert estimates (when one or other effect is correct from the point of view of an expert) are used to verify algorithms of photorealistic rendering. For instance, pioneering paper [21] shows that an image calculated for a photo of a tiger screened by a birefringent crystal demonstrates birefringence. Similarly, in papers [10, 24] it is proposed to compare visually photos with calculated images of corresponding virtual scenes.

A more accurate approach was used in the project Cornell Box [2] to demonstrate the real performance of a radiosity algorithm, where a calculated image and a photo are compared pixel-by-pixel. However, for this it was necessary to exactly specify all materials of the scene surfaces and camera setup.

In papers [4, 29], some problems of verifying the algorithms for rendering of (semi)transparent crystals are considered, and it is proposed to organize a common database of specific tests and rules. Paper [3] gives a sketch of such a base. An example of verification from [5] is given in Figs. 3-5.

7. OTHER PROBLEMS AND ALGORITHMS

In the above-mentioned papers, additional optical effects are considered:

- Fluorescence. This is a phenomenon where a substance changes the wavelength of light. In paper [26], some fluorescent paints are simulated.
- Pleochroism. This is a phenomenon where absorption in an anisotropic medium depends on the direction and state of polarization of a ray (see [36, 37]).
- Optical activity. Some crystals (for instance, quartz) rotate the plane of ray polarization during propagation through a crystal [15, 16].

8. ACKNOWLEDGEMENTS

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Figure 4: Calculated image of a virtual scene.

Figure 3: Photo of a real scene.

Figure 5: Pixel-by pixel difference of images.