Improved Algorithm of Image Compression Based on IFS.

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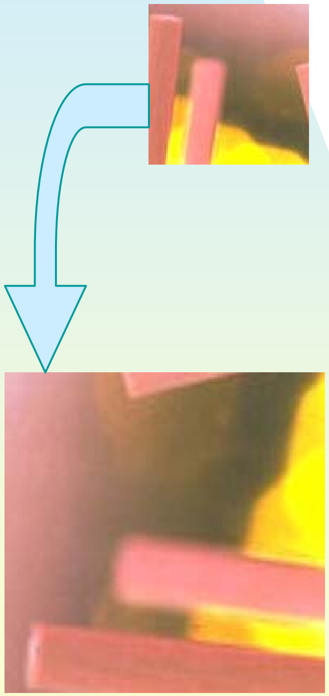
Introduction



Each encoded image in *fractal compression* is represented via *contractive mapping*.

All real implementation employ *blockwise* coding: the entire image is split on nonoverlapping block called *ranges* that tile entire image, and possibly overlapping *domains* that can be taken from any area of the original picture.

1. Mathematical background

Let us consider original signal

$$\mathbf{x} = (x_1, x_2, \ldots x_n)^T \in \mathbf{R}^n$$

and affine mapping

$$\mathrm{W} : \mathbf{x} \rightarrow \mathrm{W}(\mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{b}$$

that satisifes contractivity constraint:

$$\exists\, s \in [0,1) : d(\mathrm{W}(\mathbf{x}), \mathrm{W}(\mathbf{y})) < s\,d(\mathbf{x}, \mathbf{y})\ \forall \mathbf{x}, \mathbf{y} \in \mathbf{R}^n$$

Then exists unique fixed point

$$\exists\, \mathbf{x}_f = \mathrm{W}(\mathbf{x}_f)$$

1. Mathematical background (cont)

Encoding process is the search of **A** and **b** such that $d(\mathbf{x}_0, \mathbf{x}_f)$ is minimal.

Reconstruction of original image:

$$\mathbf{x}_f = \mathrm{W}(\mathbf{x}_f)$$

For any initial approach $\mathbf{x}_0 \in \mathbf{R}^n$

the sequence $\mathbf{x}_k = \mathrm{W}^k(\mathbf{x}_0)$

converges to the fixed point

$$\mathbf{x}_f = \lim_{k \to \infty} \mathbf{x}_k$$

1. Mathematical background (cont)

Collage theorem allows to minimize

$d(\mathbf{x}_0, \mathrm{W}(\mathbf{x}_0))$ instead of $d(\mathbf{x}_0, \mathbf{x}_f)$

Although suboptimal, this approach can be really used in practice.

2. Improved fractal algorithm

Main features

- *Independent encoding of color planes*
- *Tree-like data structure optimized for domain search*
- *Classification of range cells*
- *Hybrid algorithm that consists of:*
    - *Fast fractal encoding*
    - *Original algorithm for restoration of smooth cells*
    - *Vector Quantization (VQ) method for edge-like cells*

2. Improved fractal algorithm (cont)

"Normalizing" of domains

All domains are transformed into common representation with the same dynamic range and average color

$$\mathbf{D}^1[i] = \alpha \mathbf{D}^0[i] + \beta$$

$$\alpha = \frac{C_{max} - C_{min}}{\max\limits_{j=0,N} \mathbf{D}^0[j] - \min\limits_{k=0,N} \mathbf{D}^0[k]}$$

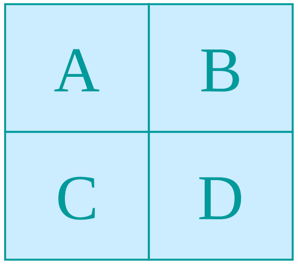
$$\beta = C_{avg} - \frac{\alpha}{N} \sum_{l=0,N} \mathbf{D}^0[l]$$

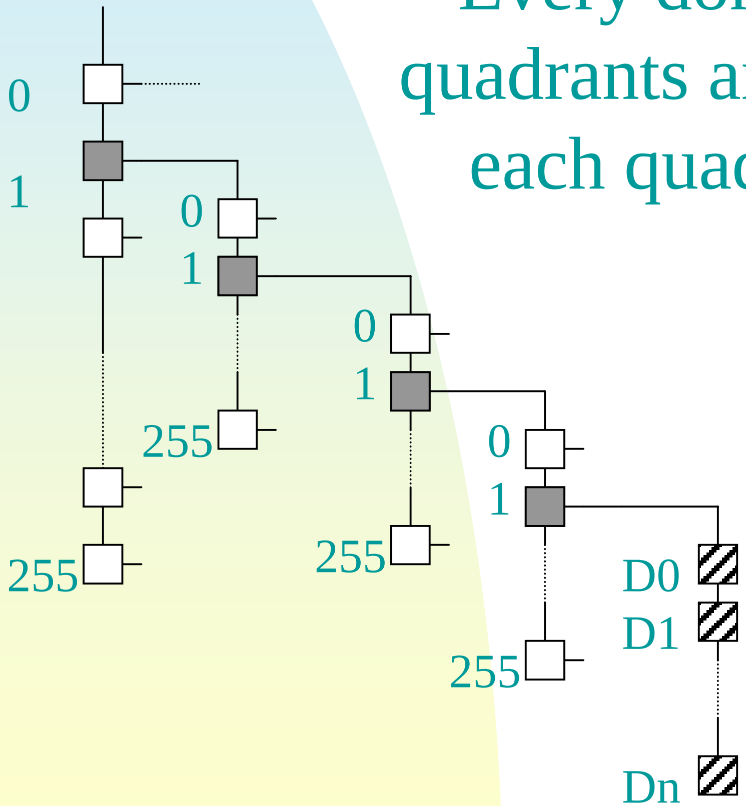
$$i = 0..N, C_{max} = 255, C_{min} = 0, C_{avg} = 128$$

2. Improved fractal algorithm (cont)

Data structure

Every domain is split into 4 quadrants and average color for each quadrant is calculated





Each of these averages is put into appropriate level of the tree. Level 5 stores numbers of Domains that have common quadrant structure. Domains are ordered by dynamic range.

2. Improved fractal algorithm (cont)

Classification of Ranges

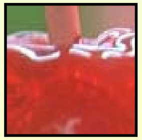
All ranges are split into 3 classes based on their dynamic range:

- *edge* (big dynamic range)

  VQ method is applied

- *smooth* (small dynamic range)

  Smooth encoding method is applied



- *midrange* (average dynamic range)

  Main fractal encoding is applied

2. Improved fractal algorithm (cont)

Domain-range matching

To find a match for a range, it is transformed:

$$d(\mathbf{R}_i, \mathbf{R}_i^D) \to \min,$$

$$\mathbf{R}_i^D[l] = \alpha I_k^i(\mathbf{D}_i^j[l]) + \beta$$

$$\alpha = \frac{\max\limits_{p=0,N} \mathbf{R}_i[p] - \min\limits_{q=0,N} \mathbf{R}_i[q]}{\max\limits_{r=0,N} \mathbf{D}_i^j[r] - \min\limits_{t=0,N} \mathbf{D}_i^j[t]}$$

$$\beta = {}^1\!/_N \sum_{u=0,N} \mathbf{R}_i[u] - {}^\alpha\!/_N \sum_{v=0,N} \mathbf{D}_i^j[v]$$

2. Improved fractal algorithm (cont)

Tree search procedure

-“Normalize” a Range, calculate quadrant averages (QA);

-For each isometry do:

-Find a match for QA1 on the first level of the tree;

-Obtain 2nd level items and repeat the same for QA2;

-Obtain 3rd level items and repeat the same for QA3;

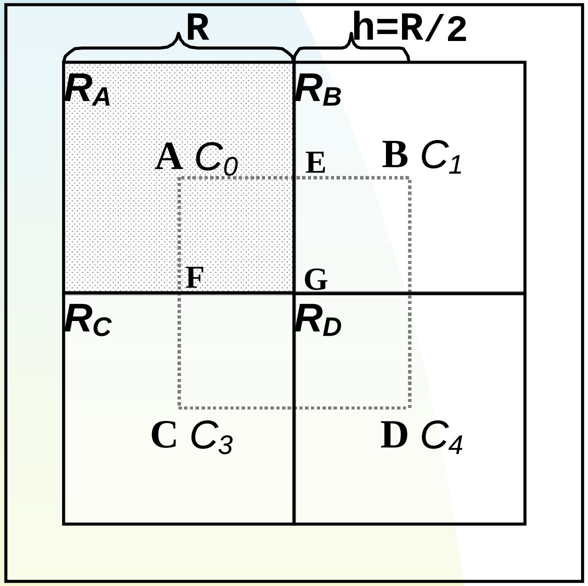
-Obtain 4th level items and repeat the same for QA4;

-Obtain list of domains and choose ones with appropriate dynamic range;

For all chosen domains calculate mapping parameters and choose one that gives minimal distortion

2. Improved fractal algorithm (cont)

Restoration of smooth areas

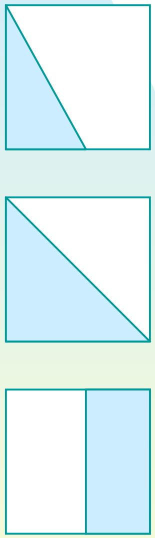


Each smooth area is divided into 4 quadrants and for each fo them linear smoothing is performed, taking into account "main" colors of smoothened area itself and neighbouring areas as well:

$$\mathbf{R}_A[i, j] = \frac{C_1 - C_0}{h} i + \frac{C_2 - C_0}{h} j + \frac{C_0 - C_1 - C_2 + C_3}{h^2} ij + C_0$$

2. Improved fractal algorithm (cont)

Vector Quantization



Our implementation uses specail set of 16 vectors that are constructed to represent well edge-like blocks of image. Each vector is 2-color; these colors take part of the fractal code built.

3. Implementation and results

Color image **600x600** is encoded in **60** sec.(about 1100 range matches per second) Range size used: **4**, distance between neighbouring domains: **8.**

Achieved compression: **15-30** times with PSNR about **31**db.

Full search would need $\mathbf{1.2\times10^8}$ range-domain comparisons ($\mathbf{1.9 \times10^9}$ "elementary" comparisons).

The proposed algorithm uses $\mathbf{4.8 \times10^6}$ "elementary" comparisons. Amount of "effective" compare operations and ones used in tree navigation are approximately equal.

3. Implementation and results (cont)

Original image

3. Implementation and results (cont)

Restored image

4. Conclusion

Presented algorithm delivers fast compression on color images.

Further improvments can be done by studying the correlation between color planes and human image recognition features.