Visualization in string theory

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Abstract
The tools for visualization of relativistic string dynamics in various topological classes are developed. New theoretical results, obtained with the aid of these tools, are described.

Keywords: Scientific visualization; Extremal surfaces; Complex dynamical systems

1 Introduction

String theory [1] is a contemporary model of elementary particles, which presents them as a system of point quarks, connected by a string-like tube of chromodynamical field. Strings have typical size 10^{-13}cm, energy about 1 GeV and a tension about 10 tons. Break of the string leads to a decay of the particle.

From mathematical point of view, string theory is a branch of differential geometry, which studies *the surfaces of extreme area* in d-dimensional Minkowsky space-time, similar by their properties to soap films in Euclidean space. In string theory such surfaces are created in motion of the string through the space-time and are called evolutionary surfaces or *the world sheets of strings.*

The aim of present work is a study of singularities, appearing on the world sheets of strings. For this purpose we combine traditional methods of differential geometry with the methods of scientific visualization. This work is a part of the project “Visualization of complex physical phenomena and mathematical objects in virtual environment”, supported by INTAS 96-0778, RFBR 98-01-00321 and 99-01-00451 grants. The results of the project have been already presented at GraphiCon’98 [2]. In compare with the previous paper [2], the sections “Strings in OpenMV Environment” and “Exotic strings” have been added.

2 Topological types of the world sheets

String theory considers the world sheets of various topological types:

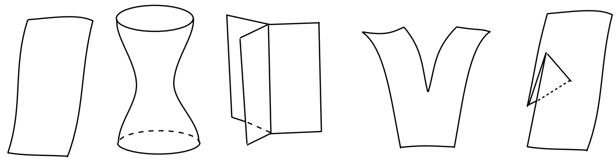

Fig.1. Main topological types of world sheets.

• open strings, the world sheet is a band in Minkowsky space, corresponds to 2-quark particles (mesons);

• closed strings, the world sheet is a cylinder, corresponds to non-quark particles (glueballs);

• Y-shaped strings, the world sheet is composed of 3 bands, glued together along one edge, corresponds to 3-quark particles (barions);

and also the surfaces of more complex topology, correspondent to transitions between the described types (decays and transmutations).

The requirement of area extremum in each topological type leads to a system of differential equations in partial derivatives, whose solutions are known in explicit form [3] and can be reproduced by the algorithms [4], convenient for visualization of string dynamics.

3 Algorithms of visualization

Reconstruction of world sheet is based on the concept of *supporting curves*. Let's consider two curves in Minkowsky space: $Q_i(\sigma) = (Q_i^0(\sigma), \vec{Q}_i(\sigma))$, $i = 1, 2$, with the following properties:

1. periodicity: $Q_i(\sigma + 2\pi) = Q_i(\sigma) + P$;

2. light-likeness: $(Q_i'^0)^2 - (\vec{Q}_i')^2 = 0$.

World sheet of closed string can be reconstructed by these curves as follows:

$x(\sigma_1, \sigma_2) = (Q_1(\sigma_1) + Q_2(\sigma_2))/2$,

i.e. as a locus of middles of segments, connecting all possible pairs of points on the supporting curves. The obtained periodical surface is topologically equivalent to a cylinder. Slices of this surface by constant time planes give the string, which is topologically equivalent to a circle and has periodically changing shape.

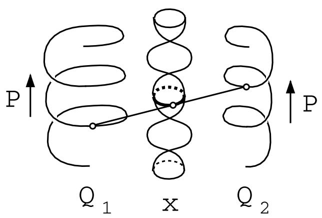

Fig.2.
World sheet of closed string.

Remarks.

1. Period P of the supporting curves in string theory coincides with the vector of total energy-momentum of the string. Subspace, orthogonal to P, forms the center-of-mass frame (CMF), where the vector of total momentum is zero. In projection to CMF supporting curves become closed.

2. There is an equivalent method for closed string reconstruction.

Let's take two *closed oriented curves of equal length* $\vec{Q}_i(\sigma)$, $i = 1, 2$, which are the projections of supporting curves in CMF. Then we mark two arbitrary points A,B on each curve and draw two arcs of equal length: AC arc along the direction of $\vec{Q}_1(\sigma)$ curve and BD arc in the direction, opposite to the orientation of $\vec{Q}_2(\sigma)$ curve. Then we connect the ends of arcs via a linear segment CD and mark its middle M.

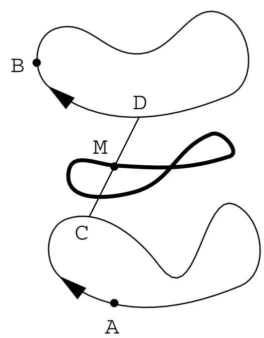

Fig.3.
Reconstruction of closed string dynamics.

While changing the length of arcs from zero to the total length of supporting curves ($L_1 = L_2 = L_{tot}$) a middle point M draws closed string at a given instant of time. Then move points A,B along the directions of corresponding curves at light velocity and repeating the described construction obtain closed string dynamics.

Formally this method corresponds to a representation of closed string:

$$\vec{x}(\tau, \sigma) = (\vec{Q}_1(\tau + \sigma) + \vec{Q}_2(\tau - \sigma))/2,$$

where $\tau \in (-\infty, +\infty)$, $\sigma \in [0, L_{tot}]$ and both curves are parametrized *naturally*, i.e. by their length: arc of the curve between points $\vec{Q}_i(0)$ and $\vec{Q}_i(\sigma)$ has length $L = \sigma$.

3. The world sheet and dynamics of open string can be reconstructed by analogous algorithms, see [5].

4. There is a necessity to consider the world sheets of the form, shown on fig.1 right, i.e. the surfaces, which topologically belong to open string class, but mapped to Minkowsky space non-trivially (with a fold). Evolution of the string can be obtained as earlier – by constant-time slicing. However, the slices of such world sheets will contain disconnected parts, and corre-

spondent evolution will include breaking/fusion of strings.

4 Implementation

For visualization of world sheets we use discretized versions of these algorithms. Constant time slicing of the world sheets, needed for animation of string dynamics, can be performed either directly (by moving clipping planes) or applying the method described in Remark 2 above. Supporting curves $\vec{Q}_i(\sigma)$ were defined by a set of 3-order piecewise-polynomial functions (*cubical splines*). The shape of supporting curve was determined by a number of control points which regulate the position, direction of tangent to the curve and its curvature. On the interval between two control points the curve was defined as:

$$\vec{Q}(t) = (-2(\vec{b}-\vec{a})+\vec{c}+\vec{d})t^3+(3(\vec{b}-\vec{a})-2\vec{c}-\vec{d})t^2 +\vec{c}t + \vec{a}, \quad t \in [0,1],$$

so that

$$\vec{Q}(0) = \vec{a},\ \vec{Q}(1) = \vec{b},\ \vec{Q}'(0) = \vec{c},\ \vec{Q}'(1) = \vec{d}.$$

On interval of $t : t \in [0, N_c]$ (where N_c is the number of control points) function $\vec{Q}(t)$ was defined as :

$$\vec{Q}(t) = \vec{Q}_i(t-i), i = [t],$$

where $\vec{Q}_i(t)$ is determined on each interval by the previous formula.

Then we should introduce natural parametrization on the curve. For this purpose we solve a differential equation :

$$\tfrac{dt}{dL} = |\vec{Q}'(t)|^{-1},$$

derived from the definition of length element $dL = |\vec{Q}'(t)|dt$. We solve this equation by 4-order Runge-Kutta method with constant step $dL = L_{tot}/N_s$, fixing the number of steps to $N_s = 160$. The obtained sequence $t_k,\ k = 0...N_s-1, t_0 = 0$ subdivides supporting curve on N_s linear segments of equal length. Subdivision points may be used for the string reconstruction by the algorithm mentioned above:

$$\vec{x}_{n,k} = (\vec{Q}_1(t_{(n+k)N_s}) + \vec{Q}_2(t_{(n-k)N_s}))/2, \quad k,n = 0...N_s-1,$$

here k is an index of a point on string, n is a discrete parameter of string evolution (frame number).

Remarks:

1. Before the integration it is necessary to define total length of supporting curve:

$$L_{tot} = \int_0^{N_c} |\vec{Q}'(t)|dt,$$

because L_{tot} determines the integration step dL. This additional integration should be performed whenever the curve is exposed to deformation.

2. This algorithm correctly reconstructs string dynamics only if steps of integration for both curves are equal: $dL_1 = dL_2$. Because total lengths of curves coincide $L_1 = L_2 = L_{tot}$, we should take equal numbers of subdivisions $N_{s1} = N_{s2} = N_s$, to satisfy the given requirement.

3. During deformation of supporting curves the equality $L_1 = L_2$ may be violated. To avoid this problem, while the first curve is deformed, the second one is scaled with coefficient L_1/L_2, that recovers the equality of lengths.

The described methods have enough speed to produce animation in real time (on machines of class SGI/O2 or higher). Two types of implementations were created:

- Java application for installation in Internet [4];
- Open Inventor and Avocado objects for installation in VR systems on SGI platform [6].

Applications take supporting curves as input data and produce string animation and 3D models of the world sheet. Supporting curves can be interactively edited and saved/loaded in a file. Using these applications, 5-min video film was also recorded, representing typical examples of string dynamics.

Additionally, relativistic string modeling was performed in frames of Open Modeler & Visualizer Environment [10–12], including such new features as visualization of energy-momentum flows on the world sheets.

Strings in OpenMV Environment.

To study and to illustrate theoretically established phenomena of relativistic string dynamics a special integrated interactive application has been developed and implemented within Open-Modeler&Visualizer (OpenMV) programming environment under UNIX/X Window. The application allows the user to reproduce different kinds of open and closed strings dynamics phenomena by means of their simulation, visualization and animation. Throgh an interface the user selects a scenario appropriate for studied problem, corrects its parameters (refines problem statement parameters, adjusts used modeling and visualization techniques, sets convenient views, places light sources, etc.), activates it and observes derived and visualized results as a animated 3D scene. The final scene is generated by means of repeated serial interpretation of the preliminary specified scenario. The interpretation can be either initiated by the user directly or caused by some changes in the scenario resulting to automatic updating final scene. This capability permits to interactively study the string problems in dependence on their statement parameters. Experienced users can compose own scenarios integrating different-purpose components for modeling and visualization to investigate more sophisticated exotic cases.

The string application has been developed as a specialization of the general-purpose OpenMV environment intended for creation of a wide range of applications for geometry modeling, physical simulation, computational mathematics, scientific visualization, computer graphics. Combining "entity-relationship" paradigm and an original object-oriented conception, the environment offers more flexibility, extensibility and reusability than traditional systems based on data flow paradigm and allows users to develop complex integrated applications for essentially different areas on the same conceptual, methodological, instrumental and programming basis.

OpenMV-based applications have a common open modular architecture that includes an object-oriented framework being invariant with respect to various areas and problems, unified graphic user interface and class libraries specific for considered applied area. Functionality of a complete application is mainly determined by included applied libraries and semantics of their classes.

According to the accepted conception, both final graphic scene and applied scenario are considered as a composition of connected typed objects. Each object has own set of specific attributes defining its internal state and behavior, set of typed links besides common properties used for its uniform manipulation. Links are external ports of objects by means of which they may connect and interact with other ones. A type of separate link predetermines potential capability of the object to connect with any other ones, types of which satisfy to the link type. Links may be single or multiple ones connecting groups of objects. To differ ways by which objects may interact via links and corresponding dependence relations arising between them, links are additionally classified as input, output and mixed ones.

We distinguish passive data-objects that can control only own behavior and active algorithm-objects that can govern behavior of other connected objects. The data and algorithm concepts express entities of arbitrary scientific data and algorithms encountered in real applications. Algorithms can change states of input objects and update states of mixed objects via links in result of activation events and performing appropriate modeling and visualization algorithms, transforms, operations, utilities.

The framework includes the *Object*, *Data*$\subset$ *Object*, *Algorithm*$\subset$ *Object*, *Scene* classes supporting the scene representation and providing needed functionality for specifying and interpreting built-in and user-defined scenarios in applications. To specify a scenario the user should define data and algorithm instances and to relate them in accordance with general inter-connectivity discipline. An interpretation of the scenario is performed by repeated serial activating algorithms that should be preliminary ordered in accordance with established dependence relations. Being sequentially activated the scenario algorithms construct new objects, update existing objects, resulting to generating final image in static mode and to its animating in dynamic mode. Once composed scenario may be then over and over again applied to study similar phenomena.

The framework is general and flexible enough to be not modified any time to suit to a particular applied problem, semantics of specific entities, their relationships, and possible details of program implementation. Realized uniform interconnecting and interaction mechanisms allow to specify and to interpret sophisticated scenarios without any refinement of semantics of concrete entities encountered in applications.

Basing on the similar principles the OpenMV graphic user interface provides unified dialogs for specification and interpretation of composed scenarios, 3D scene view windows, menus and toolbars. For clearness and usage convenience the hierarchy of registered applied classes, current scenario diagram and filtered list of scenario objects are displayed in the interfacing elements. To construct, to edit and to connect a separate object within the scenario the user sets desirable values of their publicly specified attributes and links through an appropriate unified edition dialog instance. Performed unification of the interface does not hinder its specialization for particular purposes.

Therefore, in most cases creation of a new complete application can be reduced to development of an applied class library for representation and realization of scientific data, modeling and visualization techniques specific for a particular area. Because of standard OpenMV version is oriented on scientific visualization of computational mechanics problems and provides advanced libraries for scientific data, such as structured and unstructured surface and volume meshes, polylines, point and glyph sets, scalar and vector fields, color palettes, scales, lights, view cameras as well as for visualization of fields by means of isoline and isosurface extraction, construction of streamline and streamtubes, tracing partice trajectories, pseudo-coloring, the string application has been developed as an evolution of the standard version extended by applied class library for string dynamics problems.

The developed string dynamics library includes special and general-purpose algorithm classes intended for

- construction of open and closed strings with energy and momentum distribution (specified by an explicit formula, given in [5]) by arbitrary geometry supporting curves,
- expansion of structured quadrilateral surface meshes by inclusion of bands of polyline-based cells (for reconstruction of world sheets by derived open and closed strings),
- construction of open and closed strings world sheets with energy and momentum distribution intermediately from given supporting curves,
- integrating mathematical functions by Runge-Kutta implicit methods, and auxiliary data classes for:
  - *transforming energy and momentum field distribution defined on geometry curves into mathematical functions,*
  - *manipulation with NURBS as supporting curves.*

The scenario specifies a composition of data and algorithms allowing the user to set and edit supporting curves, to construct open and closed strings, to reconstruct appropriate world sheets and to visualize obtained results simultaneously in several views as animated 3D geometry scenes pseudo-colored in accordance with corresponding energy and momentum distributions. The user can easily modify the scenario by changing values of objects attributes, by exchanging separate modeling and visualization techniques instances by other ones or even by recomposing the whole fragments to suit it to particular purposes.

Colour plate 3 shows typical examles of world sheets with density of energy colour-coded and flow of momentum shown by arrows, constructed with the aid of OpenMV Environment.

Further we list several classification theorems, characterizing string dynamics. They were initially formulated *as hypotheses* in visual study of string dynamics with the aid of the developed tools, and then proven by analytical methods.

5 Results

Singular points. In Minkowsky spaces with a dimension $d = 3, 4$ world sheets have topologically stable singular points [1]. Singularities have the following canonical forms:

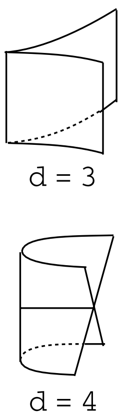

Fig.4. Canonical forms.

$\underline{d = 3}$: $(t, x, y) = (v, v + u^3, u^2)$. Constant time slices of this surface represent a cusp $(0, u^3, u^2)$, which moves in Minkowsky space in light-like direction $(1, 1, 0)$. In other words, cusp moves at light velocity perpendicularly to its own direction $(0, 0, 1)$.

$\underline{d = 4}$: $(t, x, y, z) = (v, v + u^3, u^2, vu)$. Constant time slices give a curve, which is smooth at $t \neq 0$ and has a cusp $(0, u^3, u^2, 0)$ at the instant of time $t = 0$. Projections to 3-dimensional subspaces transform this singularity is to *pinch point*, around which the surface has a canonical form of *Whitney umbrella*: (v, u^2, vu).

Remarks:

1. 3-dimensional projections of the world sheet from 4-dimensional Minkowsky space are shown on color plate 1. Two types of singular points can be found in this figure: $P, P', ...$ – singular points, existing on the world sheet itself, which are projected to pinch points in projections to 3-dimensional space (such as (xyz), (xyt), shown in the figure); Q – pinch point, which appears only on a specific projection (and therefore is not physically important).

[1] For detailed proof see [5].

2. World sheets can also have stable self-intersections, whose properties are the same as for surfaces of general form. The following tables summarize all stable singularities for the world sheets and general surfaces.

Self-intersections

	$d = 3$	$d = 4$	$d > 4$
General surfaces, world sheets	lines	points	—

Others

	$d = 3$	$d = 4$	$d > 4$
General surfaces	pinch points	—	—
World sheets	cusp lines	pinch points	—

Global behavior of singularities.

$d = 4$ *case:* stable singularities are pinch points, periodically located on the world sheet. In evolution instantaneous cusps appear on the string in its passage through the pinch points, periodically at the same place in CMF.

$d = 3$ *case* corresponds to quite complex dynamics of singularities [2]. Further consideration will be done in CMF.

Theorem 1 (presence of singularities) [7]: all world sheets of closed string in 3-dimensional Minkowsky space necessarily have singularities. In general position singularities have a form of cusp lines.

Definition:
topological charge of cusp is a number c, equal to $+1$, if a rotation from $\vec{v}$ to $\vec{k}$ is *counterclockwise*; and equal to -1 if this rotation is clockwise.

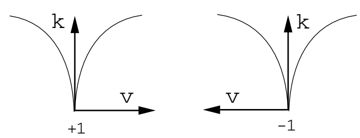

Fig.5. Topological charge.

Theorem 2
(conservation of topological charge) [4]:

[2] In string theory $d = 3$ case is used as a testbed for solution of quantum problems. It is also considered in connection with other theoretical models, e.g. *Liouville model* [7].

Total topological charge of the string, equal to the sum of topological charges of all cusps, is constant in time and equals $n_1 + n_2$. Here n_i are the numbers of revolutions of vectors $\vec{Q}_i'(\sigma)$ in complete passage around supporting curves ($n_i > 0$, if revolutions are counterclockwise; $n_i < 0$, if they are clockwise). In other words:

(number of cusps with $c = +1$)
minus
(number of cusps with $c = -1$)
equals $(n_1 + n_2)$.

Theorem 3 (permanent regime) [4]:

Let supporting curves $\vec{Q}_i(\sigma)$ have no inflection points, i.e. their tangent vectors do not change the direction of rotation during the passage. Let $\mathrm{sign}\, n_1 = \mathrm{sign}\, n_2$. In this case topological charges of all cusps are have the same sign (equal to $\mathrm{sign}\, n_{1,2}$).

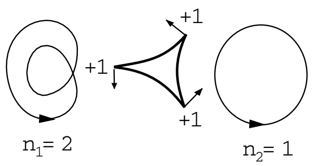

Fig.6. Permanent regime.

As a consequence, *the number of cusps* on the string is constant in time (and equals $|n_1 + n_2|$). Further consideration shows, that under permanent conditions cusps cannot collide.

Violation of permanent conditions leads to the processes of creation/annihilation of cusps. Cusps are created by pairs. At the moment of creation cusps have equal velocities and opposite directions, so that conservation of topological charge is valid: $(+1, -1) \leftrightarrow 0$.

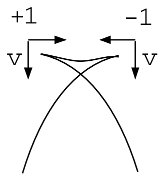

Fig.7. Creation of a pair.

Remark: curves with $n_i = 0$ (e.g. shown on fig.8) necessarily have inflection points and violate permanent conditions. Thus, these conditions imply $|n_i| \geq 1$, and the strings under permanent conditions always have $N \geq 2$ cusps.

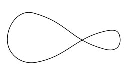
Fig.8. Curve with $n = 0$.

Analogous classification theorems are proven for open strings, see [5].

Exotic strings. Right image on fig.1 represents simplified model of exotic world sheet. Realistic image is given on color plate 2. On this figure (cABh) is a supporting curve, which has two cusps A,B. These cusps induce cusp lines on the world sheet: (fRAd) and (gBRe), which separate the world sheet into a number of pieces.

Equal-time slices of this world sheet contain disconnected parts. There is a long string, which is permanently present in the system. Additionally, the following processes occur:

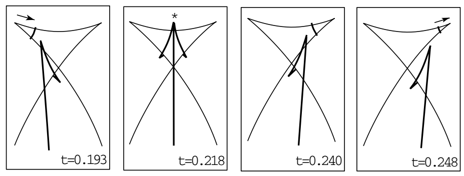

Fig.9. Exotic strings.

- at $t = 0.171$ new short string appears from vacuum;

- at $t = 0.218$ it recombinates with the long string: is attached to the long string while a part of the long string is detached;

- at $t = 0.265$ short string disappears.

Further analysis [8] shows, that linear density of energy on the string is not everywhere positive (this density changes sign in passage through the cusp lines on the world sheet, see color plate 2, where the signs of energy are shown). Components of the string, which appear/disappear in vacuum, have zero total energy, momentum and angular moment. As a result, conservation laws do not prevent such processes.

Remark. Spontaneous creation of string pairs from empty space means instability of vacuum state in string theory, predicted earlier in work [9].

6 Conclusion

Tools for visualization of string dynamics in open, closed and exotic topological classes are developed. Using these tools and analytical methods, it is shown that string dynamics in 3- and 4-dimensional Minkowsky space contains stable singularities, possessing a complex behavior. Classification theorems are proven, describing the main features of this behavior.

Further direction of the project will be investigation of string dynamics in other topological classes.

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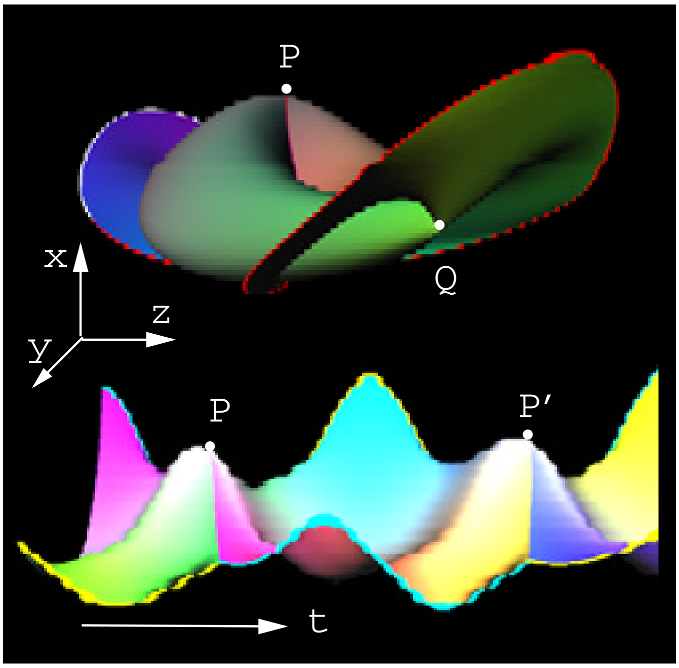
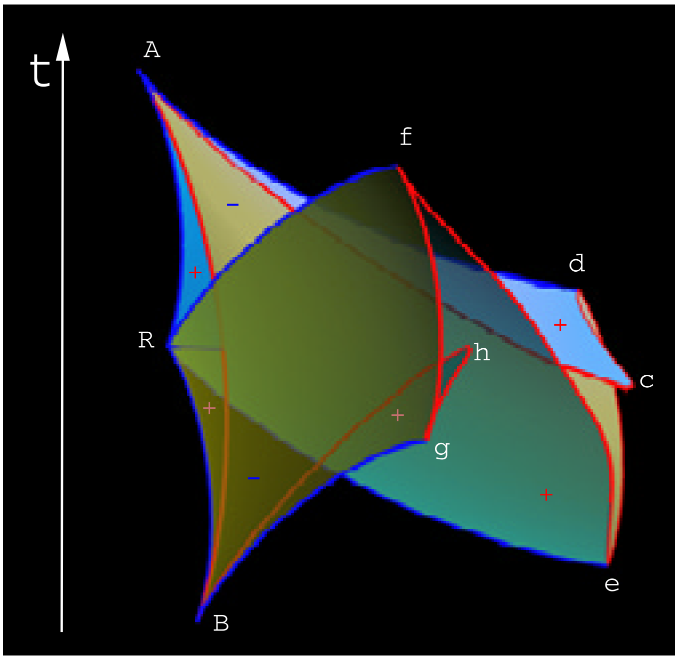
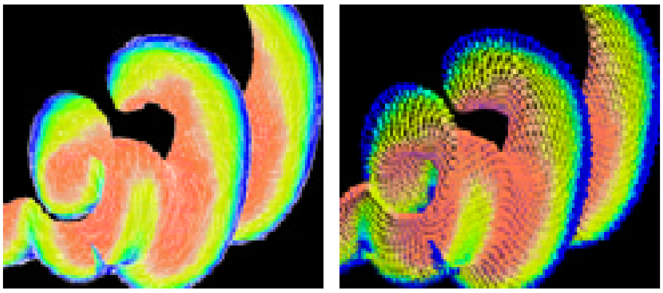


Color plate 1: Singularities on the world sheet in $d = 4$.



Color plate 2: Fold on the world sheet, typical for exotic solutions.



Color plate 3: Energetic flows on the world sheet.