Texture Parameterization With Hermite Functions

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Abstract

The report considers the task of texture identification using Hermite expansion method. Three methods to analyze textured images in terms of one-dimensional Hermite transform are described and compared. The methods were tested with Brodatz textures and real photo. Hierarchical coding without subtractions was found as the most effective method for the Hermite transform based texture identification. The method was also applied to the problem of texture based image segmentation.

Keywords: Hermite transform, texture identification, image segmentation.

1. INTRODUCTION

Texture nowadays is an important term in the world of computer vision, graphics applications and computer modeling. Although the term “texture” is wide known, it is difficult to construct a formal definition for it. Informal definitions such as *surface property*, etc. are intuitively clear for a human, but they do not reflect any approach to the textured images analysis. So it is better to say that texture is related to periodical luminosity fluctuations in the image, which let us interpret the surface as a homogenous structure. Textures can be characterized using properties such as regularity, coarseness, contrast and directionality. A system that is required to deliver meaningful judgments concerning texture, needs to be able to extract a description of the image data in a form that explicitly captures these properties.

The task of texture processing first came up with the task of making the automatic image recognition systems. Textures in the context of image recognition can be used in different ways: image segmentation; detecting the physical properties and materials of the surfaces represented in the image; detecting the shape and the position of the objects and surfaces in the image.

Our first aim will be the task of texture identification. During the past decades there have been presented different approaches [2]: statistical, geometrical, structural, model-based and signal processing. The focus of this paper is to present a new approach in the concept of the signal processing methods in texture analysis. The approach can also be referred to as filtering approach.

The signal processing method is based on applying a set of band-pass filters to extract the so-called *feature vectors* from the image. Each filter responds most strongly to a selected spatial-frequency and orientation bands. The 2-D Gabor filters *h(x,y)* are very popular to be applied to images[5],[1]:

$$h(x,y) = g(x',y')\cdot \exp(2\pi jfx'),$$

$$g(x',y') = \frac{1}{2\pi\lambda\sigma^2}\exp\left(-\frac{(x'/\lambda)^2 + y'^2}{2\sigma^2}\right),$$

$$(x',y') = (x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta).$$

These functions are often used because they are oriented, have the easy tunable carrier frequency *f*, and are good localized both in the spatial and frequency domains [1].

2. HERMITE FUNCTIONS DEFINITION

In this paper the Hermite functions were taken as an alternative to the Gabor functions. These functions are defined as:

$$\psi_n(x) = \frac{(-1)^n e^{x^2/2}}{\sqrt{2^n n!\sqrt{\pi}}}\cdot\frac{d^n(e^{-x^2})}{dx^n},$$

They also can be evaluated recursively:

$$\psi_0 = \frac{1}{\sqrt[4]{\pi}}\cdot e^{-x^2/2}$$

$$\psi_1 = \frac{\sqrt{2}x}{\sqrt[4]{\pi}}\cdot e^{-x^2/2}$$

$$\psi_n = x\sqrt{\frac{2}{n}}\cdot\psi_{n-1} - \sqrt{\frac{n-1}{n}}\cdot\psi_{n-2}, \forall n \ge 2$$

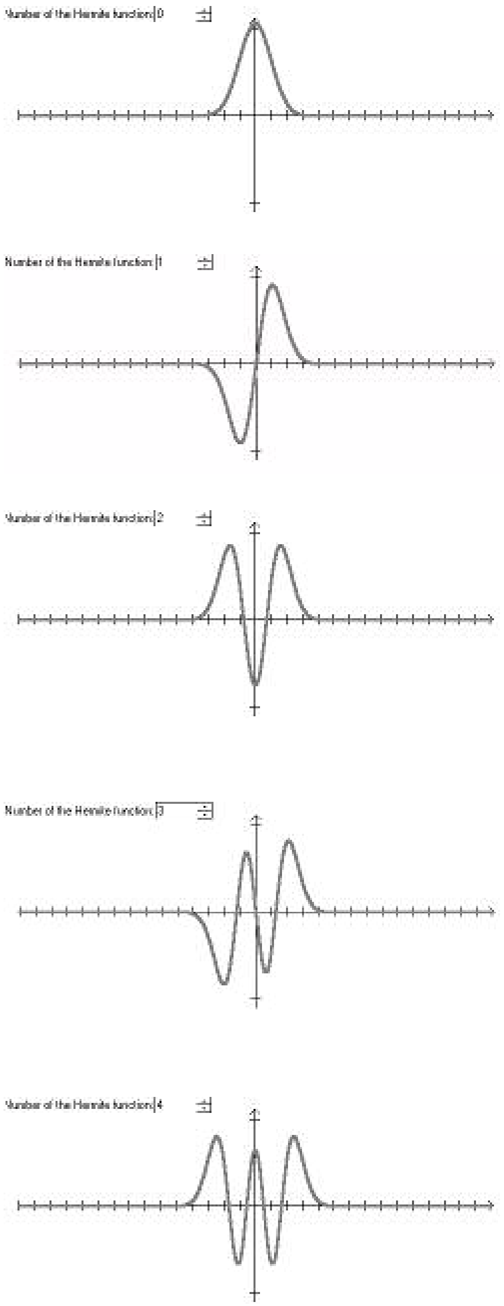
These functions vanish at infinity and satisfy the equation

$$\psi_n'' + \left(\lambda - x^2\right)\cdot\psi_n = 0, \forall\lambda = 2n+1,$$

and are the eigenfunctions of the Fourier transform.

The Hermite functions also satisfy an important feature in image processing, as they derivate a full orthonormal in $L_2(-\infty;\ \infty)$ system of functions.

Hermite functions of the first six orders are plotted in fig.1. Different applications of Hermite functions in image processing can be found in [6-10].



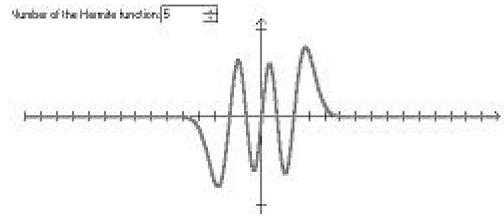


Fig.1

Now we describe the scheme of the feature vectors retrieval. We consider the 1-d function $f(x)$. It can be expanded into the series

$$f(x) = \sum_{i=0}^{\infty} \alpha_i \cdot \Psi_i(x)$$

$$\alpha_i = \int_{-\infty}^{\infty} \Psi_i(x) \cdot f(x)dx$$

where α_i is the i-th Fourier coefficient, and $\Psi_i(x)$ is the i-th Hermite function.

The 2-d Hermite function can be introduced as a separable product of the corresponding 1-d functions:
but in this current work we use

$$\psi_{n_1 n_2}(x, y) = \psi_{n_1}(x) \cdot \psi_{n_2}(y),$$

$$\psi_n(x, y) = \psi_n(x) \cdot 1$$

3. DESCRIPTION OF THE METHOD

In this paper we will describe and compare three methods to analyze textured images in terms of Hermite transform. They will be referred to as the ordinary transform, the hierarchical coding with subtractions and the hierarchical coding without subtractions.

For n=4 and n=15 the functions are plotted in fig.2

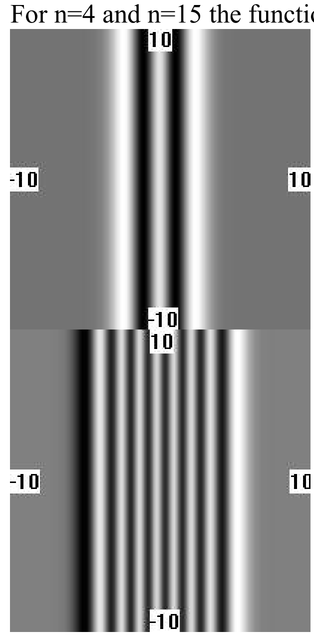


Fig.2

As can be seen in fig.2, the functions have a strongly expressed orientation. To perform an adequate analysis the functions should be turned at several different angles. In this work 8 orientation angles were used: 0, 22.5, 45, 67.5, 90, 112.5, 135, 157.5. Applying of this method to Gabor functions is described in [5].

3.1 Standard Coding

In this approach to get the feature vectors we consider the functions $\psi_n(x,y)$ where n1=0..64, and 6 energy coefficients are caclulated:

$$E_1 = (\alpha_0)^2 + (\alpha_1)^2,$$
$$E_2 = (\alpha_2)^2+(\alpha_3)^2 + (\alpha_4)^2,$$
$$E_3= (\alpha_5)^2 +(\alpha_6)^2+(\alpha_7)^2+(\alpha_8)^2,$$
$$\ldots$$
$$E_6 = (\alpha_{33})^2+(\alpha_{34})^2 + \ldots + (\alpha_{63})^2 +(\alpha_{64})^2 ,$$

f(x,y) is the source image.

Hence with the 8 orientations we get a 48-dimensional feature vector for each texture example.

3.2 Hierarchical Coding

When decomposing a function by the standard scheme in context of the texture analysis some problem arise. The most significant of them is that the images from the real world are unstructured, i.e. luminosity, scale, size, perspective of objects vary, and can cause inaccuracy during computer processing. Discussing the task of texture retrieval the problem can be interpreted as follows: human can easily detect a bricked wall in an image while variations in scale, size, perspectives can make the task unsolvable for computer vision. To avoid the mentioned problem we took the Martens' idea of the Hierarchical coding[3].

This approach lets us use several sets of patterns, inspired by the different features that can arise in an image[3]. These sets distinguish in spatial scale. So the basis Hermite functions with changing of spatial scale repeatedly take part in forming the different coefficients of the feature vector. The scheme of calculating the feature vector components is as follows:

$$E_1 = (\alpha_0^{(1)})^2 + (\alpha_1^{(1)})^2,$$
$$E_2 = (\alpha_0^{(2)})^2+(\alpha_1^{(2)})^2 + (\alpha_2^{(2)})^2 +(\alpha_3^{(2)})^2 ,$$
$$\ldots$$
$$E_6 = (\alpha_0^{(6)})^2+(\alpha_1^{(6)})^2 + \ldots + (\alpha_{62}^{(6)})^2 +(\alpha_{63}^{(6)})^2 ,$$

$$\alpha_i^{(1)} = \frac{1}{\sqrt{A_j}} \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} \psi_i(x,y) \cdot f(x,y) dx$$

$$\alpha_i^{(j)} = \frac{1}{\sqrt{A_j}} \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} \psi_i(x,y) \cdot \left(f(x,y) - \sum_{l=2}^{j} f^{(l-1)}(x,y)\right) dx, j > 1$$

f(x) is the source image,

$f^{(i-1)}(x)= \alpha_0^{(i-1)} \cdot \psi_0(x) + \ldots + \alpha_k^{(i-1)} \cdot \psi_k(x)$ is the source function's approximation at step *i-1*; $k=2^{(i-1)}-1$.

As the maximum order of the Hermite function in the current scheme is 63 we say that we have a 6-level hierarchy. An important feature in the hierarchical coding is that at each level of the hierarchy we stretch the segment of the functions' approximation to the segment $[-A_j; A_j]$ where $A_j$is found from the condition

$$\int_{-A_j}^{A_j} \psi_k^2(x) dx \geq 0.9999,$$

where *k* is the highest Hermite function order at the current

$$\alpha_i = \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} \psi_i(x,y) \cdot f(x,y) dx,$$

level. We can say that we rescale our source function to be localized at the same segment as the mentioned Hermite function. This approach makes the method of texture detection invariant to the sizes of the objects in the source textured images.

Similarly to the ordinary coding, combining with 8 orientations we get a 48-dimensional feature vector.

3.3 Hierarchical Coding Without Subtractions

This approach is the same as the hierarchical coding, but the subtraction is not performed. So we get the following feature vector:

$$E_1 = (\alpha_0^{(1)})^2 + (\alpha_1^{(1)})^2,$$
$$E_2 = (\alpha_0^{(2)})^2+(\alpha_1^{(2)})^2 + (\alpha_2^{(2)})^2 +(\alpha_3^{(2)})^2 ,$$
$$\ldots$$
$$E_6 = (\alpha_0^{(6)})^2+(\alpha_1^{(6)})^2 + \ldots + (\alpha_{62}^{(6)})^2 +(\alpha_{63}^{(6)})^2 ,$$

$$\alpha_i^{(1)} = \frac{1}{\sqrt{A_j}} \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} \psi_i(x,y) \cdot f(x,y) dx$$

$$\alpha_i^{(j)} = \frac{1}{\sqrt{A_j}} \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} \psi_i(x,y) \cdot f(x,y) dx, j > 1$$

f(x) is the source image.

Combining with 8 orientations, we get a 48-dimensional feature vector.

The results of applying these methods to some textured images can be seen in fig.3

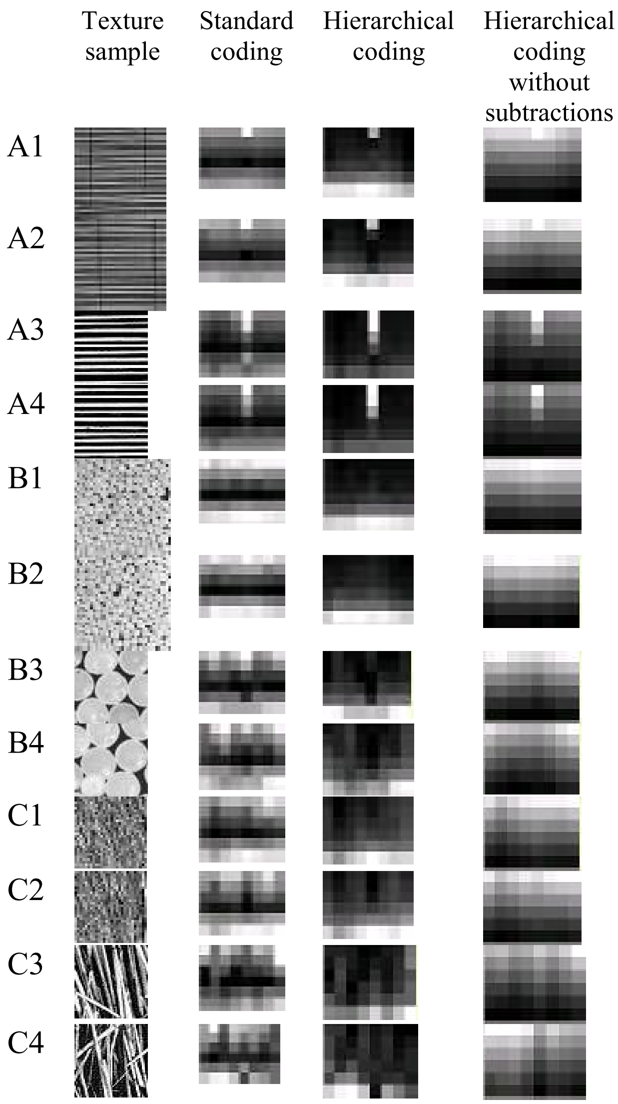


Fig.3 Structures of different textures give different patterns in the 2-D representations of the feature vectors. In the 2-D representations, spatial frequency levels increase from bottom up along the vertical axis, and orientation changes from left to right along the horizontal axis and wraps around.

4. RESULTS

To compare the resulting feature vectors we define the difference between two texture examples as

$$\rho(a,b) = \frac{\sum_{i=1}^{48}(\alpha_i - \beta_i)^2}{\max\{\sum_{i=1}^{48}\alpha_i^2, \sum_{i=1}^{48}\beta_i^2\}},$$

a, b are the two textures, α_i, β_i are the components of the feature vectors of *a* and *b* respectively.

In table1 some results are reflected:

a	*b*	Standard coding	Hierarchical coding	Hierarchical coding without subtractions
a1	a2	0.004505	0.005349	0.003739
b1	b2	0.009905	0.008160	0.007742
c1	c2	0.017778	0.011848	0.009946
a3	a4	0.008807	0.006047	0.002422
b3	b4	0.020075	0.010667	0.006275
c3	c4	0.128322	0.101591	0.080176
a1	b1	0.488420	0.492238	0.491653
b1	c1	0.315644	0.304597	0.305442

Table1. The differences between the *a* and *b* texture samples in different coding schemes.

The results in the Table 1 show that the hierarchical coding without subtractions algorithm gives better results for the Brodatz texture samples.

5. RESULTS OF APPLYING TO THE TEXTURE SEGMENTATION TASK

The proposed hierarchical coding without subtractions algorithm to obtain texture feature vectors has been tested with images of natural scenes to perform the texture-based image segmentation. All images used were 8-bit grayscale. To perform the analysis the images were split into 50x50(fig.4) and 20x20(fig.5) square blocks. For each square block the texture feature vector was calculated and by comparing them we found the regions of the similar textures. Some of the results are shown in figs.4-5 (fig.4 presents an artificial textured image with blocks of Brodatz texture samples and fig.5 is a natural scene image). Both the fig.4 and fig.5 contain the source image and two segmented images. The threshold parameter of the texture similarity was chosen manually. The “b” images were segmented with a big threshold parameter and the “c” images were segmented with approximately 10 times smaller threshold parameter. We must also notice that depending on the current image set (i.e. air photography) the threshold parameter can be obtained automatically. As is well seen from the presented illustrations our method gives adequate results for the images of natural scenes.

6. CONCLUSIONS

The Hermite coding schemes can be effectively used to analyze textured images. Further work is going on to a) design the feature vectors which are obtained using the 2-d Hermite functions and b) apply and fasten the algorithm for the pattern recognition and image segmentation tasks.

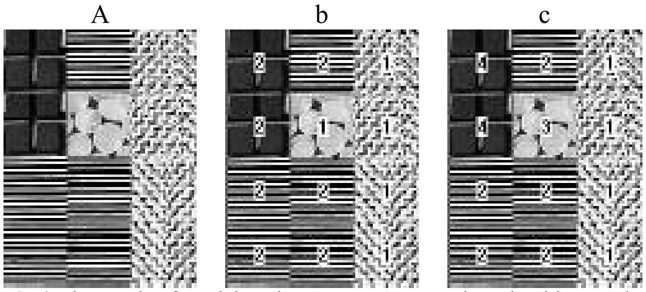


Fig.4 The result of applying the texture segmentation algorithm to the artificial textured image. With the small threshold parameter all the different textures are separated correctly (image “c”).

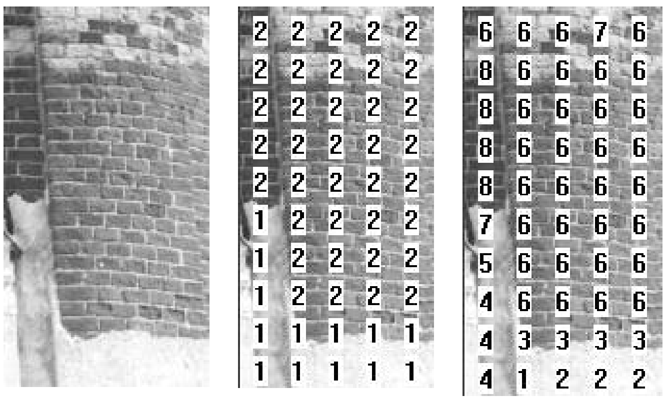


Fig.5 The result of applying the texture segmentation algorithm to the “bricked wall” image. The “strongly bricked” part of the wall is found as a homogenous surface fragment in both the “b” and “c” images, and the “c” image is separated more strictly due to the more precise threshold parameter selection.

7. REFERENCES

1. Alan Conrad Bovik, Marianna Clark, Wilson S. Geisler “Multichannel Texture Analysis Using Localized Spatial Filters”. IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 12, NO. 1, JANUARY 1990.
2. Trygve Randen, John Hakon Husoy “Filtering For Texture Classification: A Comparative Study” IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 21, NO. 4, APRIL 1999.
3. Jean-Bernard Martens “The Hermite Transform – Applications” IEEE TRANSACTIONS ON ACOUSTICS, SPEECH AND SIGNAL PROCESSING, VOL. 38, NO. 9, SEPTEMBER 1990.
4. Jean-Bernard Martens “The Hermite Transform – Theory” IEEE TRANSACTIONS ON ACOUSTICS, SPEECH AND SIGNAL PROCESSING, VOL. 38, NO. 9, SEPTEMBER 1990.
5. S. Y. Lai and W. K. Leow, “Invariant texture matching for content-based image retrieval”, In *Proc. MMM*, 53-68, 1997.
6. Y.V. Venkatesh, “Generalized Hermite polynomials for image reconstruction from zero crossing contours”, presented at the 1998 International Conference on Image Processing, October 4 - 7, 1998, Chicago, Illinois, USA.
7. Alexandre Refregier, “Shapelets: I. A method for Image Analysis” submitted to MNRAS, preprint astro-ph /0105178,2001
8. Antoon M. van Dijk, Jean-Bernard Martens, “Feature-based image compression with steered Hermite transforms”
9. P. Kruizinga, N. Petkov, “Grating sel operator features for oriented texture segmentation”, Appeared in: Proc. Of the 14th Int. Conf. On Pattern Recognition, Brisbane, Australia, August 16-20, 1998.
10. Jean-Bernard Martens, “Local Orientation Analysis in Images by Means of the Hermite Transform”, IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 6, NO. 8, August 1997.

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