Statistical Error Model of Active Triangulation Method for CAI

Iouri Volodine
Department of Robotics and Complex Automation, CAD Systems
Moscow State Technical University n.a. N.E. Bauman, Moscow, Russia
yanoscuni@mail.ru

Abstract

We develop a statistical error model for reliable computer-aided inspection (CAI) of industrial parts with the use of active triangulation optical scanning system. The work devoted to statistical analysis and accuracy evaluation of algorithms of the method. We start from obtaining an anisotropic error field for triangulation and use it as a base for further algorithms analysis. Then we construct statistically optimal algorithms using the error model of input data. We show the evolution of error through all stages of the processing pipeline. The error model of the method allows us to estimate the achieved accuracy. The use of error modeling for tolerance control makes the results more reliable and allows giving directions how to improve the accuracy of the measurements.

Keywords: *active triangulation, error modeling, anisotropic noise, tolerance control.*

1. INTRODUCTION

The need of inspection stage is crucial for many kinds of industrial production. If earlier it only related to the fields where the highly accurate and reliable parts were required, today it is a common question for production processes of a wide spectrum of manufactured goods. A traditional and time-proved method for that is the use of coordinate measuring machines. However, their manufacturing and exploitation is usually of a high cost due to the need of high-precision mechanical fixtures. Probably one of the most constraining functional parameters of CMM's is their low measurement speed which could easily lead to unacceptably great scanning time especially when the high-resolution data are requested. An alternative for contact CMM's is optical scanning. Existing optical systems could provide fast and accurate surface measurements. Although the precision of optical systems is generally lower than CMM's, under conditions it could often be sufficient for parts validation in many applications. In order to certify if the measurements accuracy is adequate to validated tolerance, the error model of the whole measurement system should be considered. This includes the physical model of the sensor and error model of processing algorithms of the method as further processing of sensed data is usually needed. The lack of thorough error modeling for the system as well as clearly defined rules for inspection with optical scanners leads to the slow industry acceptance of such equipment. Currently, there are no standards on acceptance and reverification tests for optical scanning systems as they exist for coordinate measuring machines.

On the other hand, the error modeling is necessary for well-grounded algorithms development. All algorithms on any stage of the processing pipeline starting at processing signals from sensor hardware have to deal with "real" data corrupted by errors. The fact that the errors in different parameters can be of individual size or even be correlated makes a statistical analysis of an algorithm absolutely necessary. If optimization algorithm doesn't take care of probabilistic nature of input data, then it will be difficult to estimate the resulting accuracy and to assure some accuracy level. Indeed, the accuracy of such algorithm may be far from optimal.

1.1 Previous work

One of the well-known methods is an active triangulation method. It attracted an attention of many researchers [1,2]. However, works were mainly concentrated on the measurement principle. Up to now the complete error analysis for the method is not available and just separate works devoted to that could be found. For example, some algorithms (such as triangulation [3], calibration [4,5] and so forth) were the subject of detailed study, but generally without introducing the error models. The most of early works were concentrated at development of algorithms fast enough and robust to operate in real world situations. Only few works have addressed the issues of varying or anisotropic noise. The first proposals of accuracy evaluation contained the magnitude of errors [6]. Knowing only noise magnitude implies that there is no knowledge about spatial orientation of uncertainty while for an anisotropic noise field it is rather important. The more or less comprehensive model for active triangulation system can be found in [7]. The authors derived a noise model for sensor based on triangulation principle but they didn't address the remaining part of processing, namely the registration, integration and tolerance verification algorithms. In this work we will follow an approach similar to one described in [7] (the first order propagation of covariance) to deduce the covariance matrices for every point of work area of the scanner.

The work [8] is going further and presents a modification of registration algorithms first appeared in [9], making use of anisotropic sensor noise model. The true worth of this work is that they did not only use the noise model of input data to construct an optimal algorithm from the point of view of chosen statistical criteria, but also performed the statistical analysis of this estimation algorithm and acquired the additional uncertainty which it adds to output data. In this paper we will reformulate their registration algorithm for purposes of computer aided inspection and also propose an evolution of closest point search algorithm by incorporating there the noise model.

1.2 Scope of the work

We will further develop a complete error model from the data acquisition stage up to the tolerance control for computer aided inspection of industrial parts with optical triangulation scanner. We will follow the error propagation through the stages of processing. That will give us the possibility to formulate the problems of creation of optimal algorithms for every stage and to choose statistically justified criteria functions.

We will start from data acquisition using active triangulation system and will derive an anisotropic error model for triangulation. For this we will take sensing and vectorization uncertainty in image coordinates and then we will propagate it to 3D coordinates of measured points. This anisotropic noise model will serve as a base for subsequent algorithms. After that we will discuss registration of several scanned views of an object. Registration is needed for eliminating the errors in sensor position and orientation. Then an approach for tolerance control will finally benefit from the uncertainty analysis on previous stages.

2. ERROR MODEL OF ACTIVE TRIANGULATION SYSTEM

An active triangulation method is the common method for acquisition of 3D coordinate data. The principle of the method is the use of stereoscopic parallax to get the information about third dimension (see Fig.1).

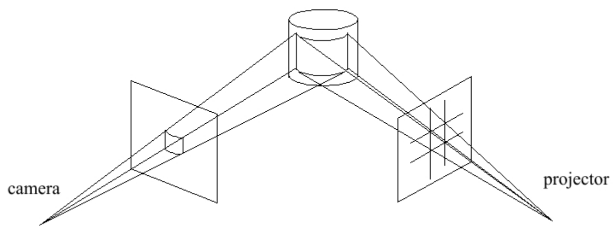

Figure 1: The principle of active triangulation.

In contrast to stereo method we use here a projector instead of second camera. The system project specific light patterns onto the object. The light patterns are distorted by the object surface and observed by the camera. The central part of 3D reconstruction is a triangulation algorithm, which will be discussed in greater detail in the next section. Here we will only illustrate that this algorithm necessarily introduces an anisotropic error field (Fig. 2).

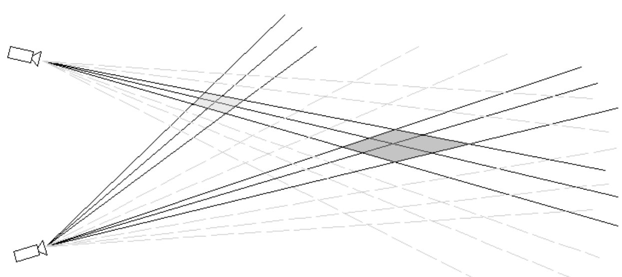

Figure 2: Illustration of anisotropic uncertainty field of triangulation method.

2.1 Triangulation

Let the point $\mathbf{P}$ in the world coordinate frame *XYZ* has coordinates $\mathbf{P} = (x \quad y \quad z \quad 1)^{\mathrm{T}}$. The image of $\mathbf{P}$ on camera registration plane is a vector $\mathbf{P}_k = (wx_k \quad wy_k \quad w)^{\mathrm{T}}$. The mapping between $\mathbf{P}$ and $\mathbf{P}_k$ is a projective transformation $\mathrm{M}_k\mathbf{P} = \mathbf{P}_k$, where M_k is a composition of matrices of rotation, translation and central projection. M_k - is a rectangular matrix of 3x4. The point $\mathbf{P}_{pr} = (vy_{pr}, v)^{\mathrm{T}}$ is a corresponding point for $\mathbf{P}$ on a slide plane of projector. The points $\mathbf{P}$ and $\mathbf{P}_{pr}$ are also related by a projective transformation $\mathrm{M}_{pr}\mathbf{P} = \mathbf{P}_{pr}$, where M_{pr} - is a rectangular matrix 2x4. In triangulation the matrices M_k and M_{pr} and coordinates of projections $\mathbf{P}_k$ and $\mathbf{P}_{pr}$ are supposed known. The problem consists in determination of 3D coordinates of point $\mathbf{P}$. Thus, in general case we can write:

$$\mathbf{P} = \tau(\mathbf{P}_k, \mathbf{P}_{pr}, \mathrm{M}_k, \mathrm{M}_{pr}), \qquad (1)$$

where τ - a triangulation method.

From the equation (1) we see that the accuracy of all triangulation parameters will affect the resulted accuracy of 3D point determination. The coordinates of projection $\mathbf{P}_k$ and $\mathbf{P}_{pr}$ are usually measured directly and the matrices M_k and M_{pr} are the subject of estimation during the calibration step. The calibration is a canonical problem in computer vision society. There are a lot of specialized methods in this field which could achieve a very high level of reliability and precision [5]. Here we will not consider issues of sensor calibration and will suppose that before the digitizing the sensor was properly calibrated and on the stage of triangulation we are aware of exact calibration matrices.

Our accuracy analysis we will base on the data on the accuracy of direct measurements of camera and projector coordinates for a point. Some numerical results were reported in work [7].

According to analytical first order propagation of covariance method (for detailed derivation see for example the work [10]) we can write:

$$\Lambda_{\mathbf{P}} = (\nabla\tau^{\mathrm{T}}\Lambda_{\mathbf{t}}^{-1}\nabla\tau)^{-1} \qquad (2)$$

where $\Lambda_{\mathbf{P}}$ - the covariation matrix of the point $\mathbf{P}$, $\Lambda_{\mathbf{t}}$ - the covariation matrix of projections, $\nabla\tau$ - the Jacobian matrix. On conditions that the measurements of $\mathbf{P}_k$ and $\mathbf{P}_{pr}$ are statistically independent the matrix $\Lambda_{\mathbf{t}}$ is a block diagonal:

$$\Lambda_{\mathbf{t}} = \begin{bmatrix} \Lambda_{\mathbf{P}k} & \mathbf{0} \\ \mathbf{0} & \Lambda_{\mathbf{P}pr} \end{bmatrix}$$

We will then suppose that some estimate of the matrix $\Lambda_{\mathbf{t}}$ is already available (it could be obtained empirically or from some constructional information) and we will deduce the formula for world point covariation.

The triangulation in this concrete case is usually done directly by solving the system of equation:

$$\begin{bmatrix} \mathrm{M}_k \\ \mathrm{M}_{pr} \end{bmatrix} \mathbf{P} = \begin{bmatrix} \mathbf{P}_k \\ \mathbf{P}_{pr} \end{bmatrix}$$

From this we get:

$$\begin{bmatrix} x_k \\ y_k \end{bmatrix} = \begin{bmatrix} \mathrm{M}_k^1\mathbf{P} \big/ \mathrm{M}_k^3\mathbf{P} \\ \mathrm{M}_k^2\mathbf{P} \big/ \mathrm{M}_k^3\mathbf{P} \end{bmatrix}, \; y_{pr} = \frac{\mathrm{M}_{pr}^1\mathbf{P}}{\mathrm{M}_{pr}^2\mathbf{P}} \qquad (3)$$

where M_k^i denotes the *i*-th row of the matrix.

Then, the derivation is straightforward. The Jacobian matrix will be:

$$\nabla\tau = \begin{bmatrix} \dfrac{\mathrm{M}_k^1\mathrm{M}_k^3\mathbf{P} - \mathrm{M}_k^3\mathrm{M}_k^1\mathbf{P}}{(\mathrm{M}_k^3\mathbf{P})^2} \\ \dfrac{\mathrm{M}_k^2\mathrm{M}_k^3\mathbf{P} - \mathrm{M}_k^3\mathrm{M}_k^2\mathbf{P}}{(\mathrm{M}_k^3\mathbf{P})^2} \\ \dfrac{\mathrm{M}_{pr}^1\mathrm{M}_{pr}^2\mathbf{P} - \mathrm{M}_{pr}^2\mathrm{M}_{pr}^1\mathbf{P}}{(\mathrm{M}_{pr}^2\mathbf{P})^2} \end{bmatrix}$$

Applying eq. (2) we get a covariance matrix $\Lambda_{\mathbf{P}}$.

The results for the working area of the sensor are presented in Fig. 3. Figure 3 shows the variation of the longest axis of uncertainty ellipsoid over the working area of the sensor.

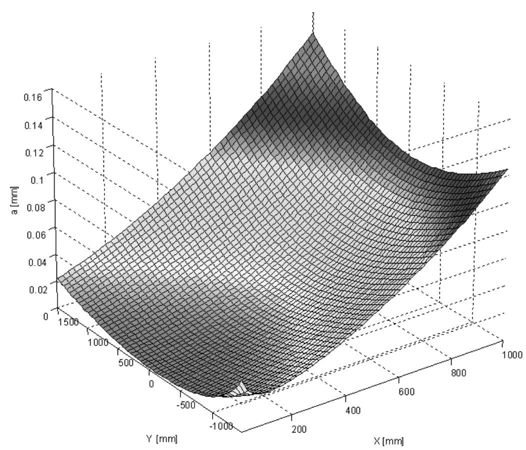

Figure 3: The variation of the longest axis of uncertainty ellipsoid over the working area of the sensor.

The use of error model of the system will give us a ground for selection of sensor placements and orientation regarding to the measured part.

3. REGISTRATION

The need for this processing step arises from errors in positioning and orientation of sensor during acquisition. From the aspect of CAI registration means the alignment of datasets of measured points from each view with the CAD-description of the scanned part. As long as the description of an object is known a priori we can perform registration for each view independently. This could be classified as a pair-wise registration problem. The major difference between this formulation and the problem addressed in [8] is that here only the measured points are known with error while the CAD-model gives exact description. It allows us to rearrange the pair-wise registration algorithm for this particular case. The algorithm we use is a modification of iterative closest point algorithm [9] with incorporated noise. The algorithm gradually iterates through the steps of closest point search and pose estimation until it converges (the change of cost function falls below established threshold). The problem of finding a transformation between dataset and CAD model is solved by determining the transformation that minimizes the sum of squared distances between corresponding point pairs. The algorithm can be sketched as follows [11]:

Algorithm I

1. Closest point search. We find the pairs of corresponding points for pose estimation. For that we take a measured point from the dataset and find the closest point of the CAD model (see 3.1 for details).
2. Pose estimation. We find the transformation which superposes the dataset points with the corresponding portions of CAD model. Due to the measurement errors the points won't coincide exactly; so we have an optimization problem and search for an optimal solution in terms of statistical criteria. The algorithm is described in detail in [8]. Our problem could be easily deduced from the more general case by incorporating noise only in one set of points. This will cast away one of components in sum of covariation matrices.
3. Dataset transformation. We apply transformation found on the previous step to point cloud.
4. Repeat 1-3 until convergence.

In contrast to [8] we apply noise model to the closest point search as well as the pose estimation problem. Next we will examine separate steps in more details.

3.1 Closest Point Search

For given point from a dataset we are eager to find a closest point on a CAD model. We are going to derive formulas for two common formats of CAD models: polygonal triangular grid model of STL format and NURBS surface model of IGES format. We bring in discussion the STL model for the reason of significantly lower computational resources required to registration. The almost constant time to find a triangle is reported by many authors [11].

3.1.1 The Closest Point on Triangle

For a closest point on triangle we will use the barycentric coordinates α, β, γ with respect to triangle ABC. Thus, the closest point to the triangle plane will be the point for which the distance $\left\| \mathbf{P} - \alpha A - \beta B - \gamma C \right\|$ is minimized. Here we have only two independent coordinates since $\gamma = 1 - (\alpha + \beta)$. We incorporate the error model by using Mahalanobis distance in minimization problem:

$$\|\mathbf{P}-\alpha A-\beta B-\gamma C\|_{\Lambda} \to \min \qquad (4)$$

where $\| \ \|_{\Lambda}$ denotes the Mahalanobis metric with covariance matrix Λ. For covariance matrix Λ we will use the matrix acquired on the current step of Algorithm I.

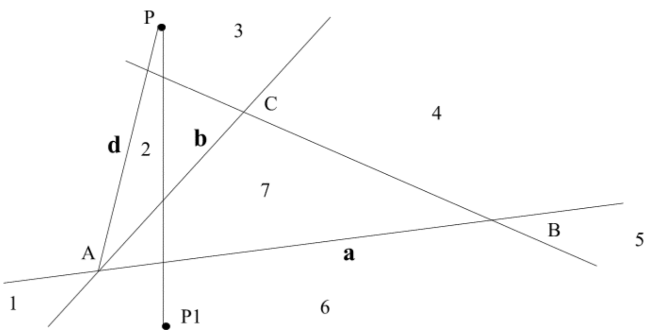

Figure 6: The closest point on triangle.

Differentiating the function in (4) we will have a linear system of 2 equations which can be solved analytically with respect to unknown α, β, γ. Then, depending on the region in which the found point $\mathbf{P}1$ lies (see Fig. 6) the algorithm will choose between $\mathbf{P}1$ (in case the point lies inside triangle) and the nearest point to $\mathbf{P}1$ but lying on the edge of triangle or coincide with one of the vertices (if it lies on the outside).

3.1.2 The Closest Point on NURBS Surface

We will extend the method of closest point search described in [12]. According to it, the closest point on NURBS surface is sought as a parameterization which delivers the solution to the following minimization problem:

$$\min_{(u,v)} \|\mathbf{P}-\mathbf{s}(u,v)\|^2,$$

where $\mathbf{s}(u,v)$ is a parametric surface. After the Taylor expansion this could be rewritten in matrix form as:

$$\min_{(u,v)} \|\mathbf{J}w-d\|^2,$$

where $\mathbf{J}$ - is the Jacobian matrix of $\mathbf{s}(u,v)$ and w is the variation of parameterization. The important extension of this approach exploiting uncertainty information is the use of Mahalanobis metric:

$$\min_{(u,v)} \|\mathbf{J}w-d\|_{\Lambda}^2,$$

The linear least square solution could then be obtained:

$$w = (\mathbf{J}^{\mathrm{T}}\Lambda^{-1}\mathbf{J})^{-1}\mathbf{J}^{\mathrm{T}}\Lambda^{-1}d \qquad (5)$$

This gives the generalization of closest point computation proposed in [12] which takes into account the presence of noise in measurements.

4. CONCLUSION

We have presented an error model for active triangulation method. We showed the evolution of uncertainty through the stages of processing up to the tolerance control. We have proposed the statistically optimal algorithms which take into consideration uncertainty data. We have illustrated the concept of using statistical information on every stage of the method. The use of error model lets improve the accuracy of measurements carried out with the help of active triangulation system. It will hopefully contribute to the growth of the field of application of optical scanners.

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About the author

Iouri Volodine is a post-graduated student at Moscow State Technical University n.a. N.E. Bauman, Department of Robotics and Complex Automation, CAD Systems. His contact email is yanoscuni@mail.ru