A new approach to Bézier surface visualization by a ray tracing method

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Problem of free – form surfaces visualization is actual in various areas of a science and engineering. One of mathematical models used for this purpose is the mathematical description of a Bézier surface [1, 2]. Classical methods of computer graphics based on polygonal models [3] use only polygonal interpolation of a Bézier surface. Such approach inevitably leads to occurrence of an error and as a consequence – to discrepancy of an image.

There were number of approaches proposed for direct ray-tracing of free-form surfaces in the last two decades.

Nishita et al. [4] describes an iterative algorithm called Bézier Clipping to compute intersections between a ray and a Bézier patch by identifying and cutting away regions of the patch that are known not to intersect the ray. Efremov et al. [5] proposed some modification to Nishita's algorithm for NURBS surfaces, but in this case the number of patches and volume of calculation increasing.

Martin et al. [6] describes a framework for ray tracing of trimmed NURBS used Newton's method. Wang et al. [7] combined Newton iteration and Bézier-clipping and used the coherence of neighboring rays to speed up Nishita's algorithm.

In this work we propose a new approach for unknown parameter's values finding, which are necessary for visualization of a Bézier surface (on an example of a biquadric surface). Usually mathematical task of a ray and surface intersection point search consist as roots finding of the nonlinear equation system. To define unknown parameters (*u* and *v*), equation system should consider a case when the ray coincides with one of coordinate axes (*Oz*). Our approach is based on transformation of a problem of a roots finding to a problem of optimization. In this case the mathematical task of a ray and a surface intersection point search can be written as optimization task. As for parameters finding the function of minimization can be described as follows:

$$w = \left(Q_X(u,v)\right)^2 + \left(Q_Y(u,v)\right)^2 \to \min_{u,v},$$

where:

$Q_X(u,v)$, $Q_Y(u,v)$ – Bezier surfaces equations.

Task of function *w* optimization should be divided into two parts: preliminary search and optimization of the parameters. For reception of preliminary values (for primary ray tracing) of parameters the map of preliminary values is made. The preliminary search for secondary ray tracing is based on a method of Monte-Carlo. For optimization we choose a gradient method [8].

On the one hand such approach allows, as against a Nishita's method, to find not one intersection point, but some points (without patch clipping). On another hand it allows to reduce volume of calculations.

In this work were implemented proposed method as well of the methods suggested by Nishita et al., Martin et al. and Wang et al.

The received results of experiment have shown, that:

- The use of our method gives the very satisfactory results on quality of the image in comparison to other methods.
- A method Nishita et al. [4] has given the worse results on quality of the image in experiment
- A method Martin et al. and a method Wang et al. these methods give identical result. The distinctions even by attentive consideration are not found. The disadvantage of methods is found on border of patches.
- Our method has not disadvantages on quality of the image as in a method Nishita et al. (distortion texture) and method Martin et al. (distortion on border of patches).
- In our experiment Use of proposed method gives the best result on image rendering's time in comparison with others methods.

The rendering of other objects (rational Bézier surfaces, NURBS et al) is subject of research in future works.

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