

# Averaging of Fingerprint Template with Respect to Elastic Deformations

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## Abstract

The paper describes a technique of generation of an “undeformed”, i.e. averaged with respect to elastic deformations, fingerprint template. We offer to calculate the average “undeformed” fingerprint from multiple impressions. The averaging is based on the elastic model of fingerprint. Experiments show that the “undeformed” template improves fingerprint identification performance as measured by the FAR and FRR.

**Key words:** *fingerprint recognition, elastic deformations, template improvement.*

## 1. INTRODUCTION

The performance of fingerprint recognition algorithms is negatively affected by different factors: noises, differences in skin conditions, elastic distortions of fingerprints (fig.1) etc. In this paper we consider elastic deformations. The aim of our research is prior compensation (during enrollment) of elastic deformations of fingerprints.

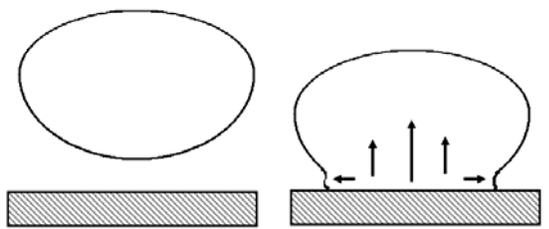


Figure 1: Origin of elastic deformations

There are different approaches to registration and modeling of elastic deformations. One of the first approaches was introduced by D.J. Burr [9], and used the concept of rubber masks. The way suggested by A.M. Bazen and S.H. Gerez [10, 11] is based on the thin-plate spline (TPS) models, firstly applied to biological objects by F.L. Bookstein [12]. This method requires determining correspondent points in two compared images (matching point) and it suffers from the lack of precision in case of few matching points. Modifications of TPS (approximate thin-plate splines and radial based function splines) were introduced by M. Fornefett, K. Rohr and H. Stiehl [13],[14]. They consider deformations of biological tissues. But this way also requires many matching points (more than 100) what is virtually impossible in fingerprint applications, because number of minutiae in fingerprint image rarely exceeds 50. This fact makes TPS and its variants hardly applicable to fingerprint deformations registration. Very interesting empirical approach has been suggested by R. Cappelli, D. Maio and D. Maltoni [15]. They developed analytical model of fingerprint deformation, however not specifying the algorithm for its registration.

The listed approaches and techniques don't provide us with direct tools for prior registration of fingerprint deformations. The only approach to prior deformation estimation has been proposed in [11]. Authors suggest that a fingerprint template can be averaged on-line. Additional information for averaging is obtained during each genuine match.

In this paper we combine averaging technique [11] and elastic model [16] of fingerprint deformation. The paper is organized as follows. Section 2 describes mathematical model of fingerprint deformations based on the elasticity theory. The technique of deformation averaging is in Section 3. Section 4 contains experimental results.

## 2. MODEL OF DEFORMATION

Let's represent a deformation of fingerprints as a mapping of images:

$$\mathbf{u} : \text{Im}_1 \rightarrow \text{Im}_2, \quad (1)$$

$$\text{Im}_2(x, y) = \text{Im}_1(x + \mathbf{u}(x, y), y + \mathbf{u}(x, y)). \quad (2)$$

where  $\text{Im}_1$ ,  $\text{Im}_2$  - fingerprint images,  $\mathbf{u}$  - displacement field.

Let's assume that fingerprint is elastic body. Dynamics of its deformation are defined by the following equations [17]:

$$\begin{aligned} \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xx}}{\partial x} + f_x &= \rho \frac{\partial^2 u_x}{\partial t^2}; \\ \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + f_y &= \rho \frac{\partial^2 u_y}{\partial t^2}; \end{aligned} \quad (3)$$

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}; \quad \varepsilon_{yy} = \frac{\partial u_y}{\partial y}; \quad \varepsilon_{xy} = \varepsilon_{yx} = \frac{1}{2} \left( \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right);$$

where  $u_x$ ,  $u_y$  are the  $x$ - and  $y$ - components of displacements,

$\mathbf{f} = (f_x, f_y)$  is external forces,  $\varepsilon$  is the strain tensor,  $\sigma$  is the stress tensor,  $\rho$  is the mass density.

Relations between tensors  $\varepsilon$  and  $\sigma$  obey Hooke's law:

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \mathbf{C} \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{pmatrix}, \quad (4)$$

$$\mathbf{C} = \frac{1}{E} \begin{pmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & (1+\nu) \end{pmatrix}. \quad (5)$$

where  $\mathbf{C}$  is the stiffness matrix,  $E$  is the Young's modulus,  $\nu$  is the Poisson's ratio.

As known from the elasticity theory [17], steady-state solution (when the fingerprint is immobile in the surface of contact with a scanner) of (3) minimizes the following energy function:

$$\begin{aligned} E &= - \iint_S (u_x f_x + u_y f_y) dx dy + \\ &+ \frac{1}{2} \iint_S (\varepsilon_{xx} \sigma_{xx} + \varepsilon_{yy} \sigma_{yy} + \varepsilon_{xy} \sigma_{xy}) dx dy. \end{aligned} \quad (6)$$

As shown in [16], the energy function can't be directly calculated from (6), because external forces  $\mathbf{f} = (f_x, f_y)$  are unknown. We offer approximate solution of (6) that based on measurable fingerprint features.

Let's assume, that we have found paired minutiae sets  $\{\mathbf{p}_i\}_{i=1}^m$  and  $\{\mathbf{q}_i\}_{i=1}^m$  on the first and second images respectively (fig.2). We replace action of forces  $\iint_S (u_x f_x + u_y f_y) dx dy$  with mean-square discrepancy:

$$L^2(\mathbf{u}) = \frac{1}{m} \sum_{i=1}^m \|\mathbf{p}_i + \mathbf{u}(\mathbf{p}_i) - \mathbf{q}_i\|^2. \quad (7)$$

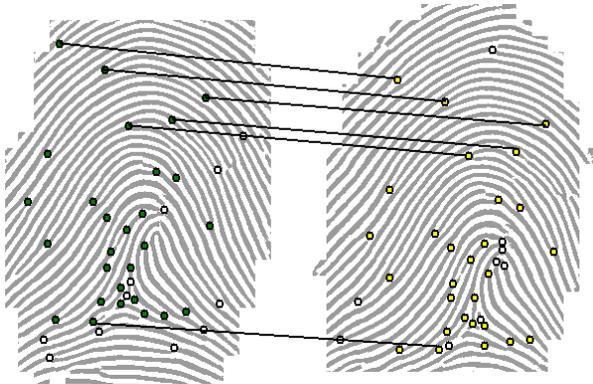


Figure 2: Minutiae correspondence

The final energy functional is a sum of the internal deformation energy and the substitute of the external forces work (7):

$$D(\mathbf{u}) = E_d(\mathbf{u}) + \alpha L^2(\mathbf{u}), \quad (8)$$

where  $\alpha$  is the weight coefficient,  $E_d$  is the internal deformation energy:

$$E_d = \frac{1}{2} \iint_S (\varepsilon_{xx} \sigma_{xx} + \varepsilon_{yy} \sigma_{yy} + \varepsilon_{xy} \sigma_{xy}) dx dy. \quad (9)$$

Minimum of (8) defines relative deformation of the first image  $\text{Im}_1$  to the second image  $\text{Im}_2$ . An example of fingerprint deformation is in the Fig. 3.

### 3. MODEL OF UNDEFORMED FINGERPRINT

Let's assume that we enrolled  $k+1$  fingerprint images:  $\text{Im}_0, \text{Im}_1, \dots, \text{Im}_k$ . Relative deformation of  $\text{Im}_j$  to  $\text{Im}_0$  is denoted as  $\mathbf{u}_j$ . Let's define an "undeformed" fingerprint image as a minimum of the following function:

$$\mathbf{u}_{\min} = \min_{\mathbf{u}} \left( \sum_{j=1}^k E_d(\mathbf{u} - \mathbf{u}_j) + E_d(\mathbf{u}) \right). \quad (10)$$

Physically the function (10) is the total energy of deformation from condition  $\mathbf{u}_{\min}$  to every impression  $\text{Im}_0, \text{Im}_1, \dots, \text{Im}_k$ . "Average" fingerprint image is result of the following mapping  $\text{Im}_{\min} = \mathbf{u}_{\min}(\text{Im}_0)$  (fig.4).

The solution of (10) depend only on positions of minutiae points in fingerprint images. We can calculate the "undeformed" condition not only for fingerprint image but also for the ISO-compliant fingerprint template. It allows adding the proposed technique on operating fingerprint identification systems.

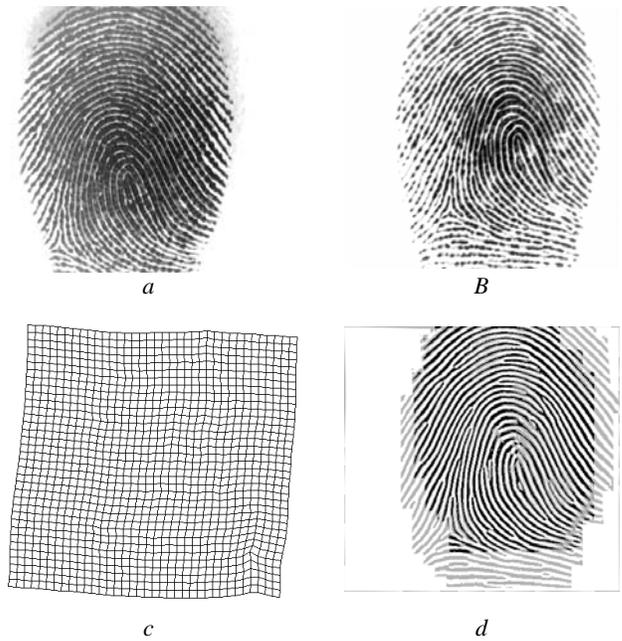


Figure 3: Example of deformation calculation (a,b – source images, c – displacement field, d- binarized image overlap).



Figure 4: Deformation averaging (undeformed image is on the right)

### 4. EXPERIMENTS

Let's briefly describe our experiments with the "averaging" of fingerprint templates. We employ the Biolink fingerprint recognition SDK and the public available FVC2002DB1 dataset [18]. The FVC2002DB1 contains 8 impressions of 100 different fingerprints (as total 800 images). Let's denote it as  $\text{Im}_j^i$ , where  $i$  is the impression index,  $j$  is the fingerprint index. We randomly select 4 impression subset  $i_1, i_2, i_3, i_4$ . The query dataset contains averaged fingerprint templates generated from  $\text{Im}_{j_1}^{i_1}, \text{Im}_{j_2}^{i_2}, \text{Im}_{j_3}^{i_3}, \text{Im}_{j_4}^{i_4}$  (100 templates). The target dataset contains 400 templates (4 impressions of 100 fingerprints). As total, we have 400 genuine and 39600 impostor matches for each subset  $i_1, i_2, i_3, i_4$ . The we measure FAR and FRR. DET curves of fingerprint recognition are in the Fig. 5. The "undeformed" fingerprint template significantly improves performance as measured by the DET.

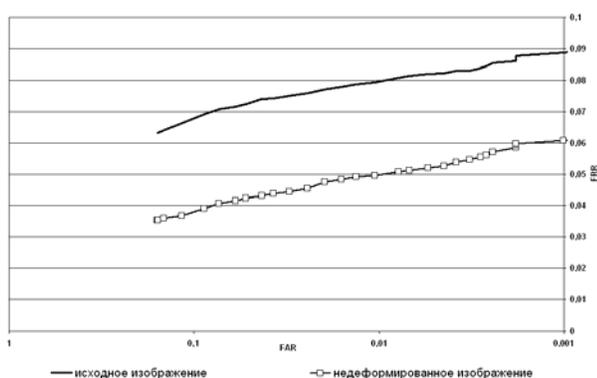


Figure 5: Performance improvement (FVC2002DB1)

## CONCLUSION

We proposed the technique of generation of “undeformed” fingerprint image and generation of “undeformed” fingerprint template. The technique allows improving fingerprint recognition performance as measured by the FAR and FRR. In further we plan fusion of our fingerprint “averaging” with mosaicking technique [19,20].

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