New Ridge Count Model based on Fingerprint Image Topology

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Abstract

The paper briefly describes a mathematical model of fingerprint based on ridge count vectors. They are computed from topological descriptors of minutiae point neighborhood and stored in the template with the list of minutiae. Templates are used to identify the fingerprint. The approach allows designing ridge count value as the fractional number.

Keywords: *Fingerprint, topology, minutiae, ridge count, fractional number.*

1. INTRODUCTION

In computerized systems (CS) verification and identification of fingerprint images (FI) usually realizes with templates. Template is a set of compact formal structures, for example, minutiae, such as bifurcation and ending. They are usually detected using skeleton lines [1–3], shown in figure 1, as a formal presentation of FI.

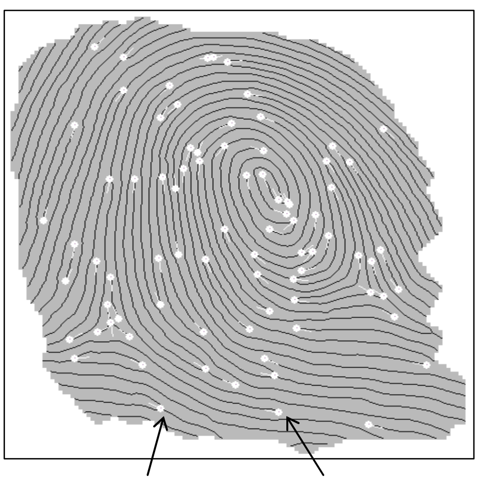

Figure 1. Skeleton and minutia of fingerprint image

Identification procedure regards the template as mathematical model of FI, which should be based on necessary and sufficient number of minutiae [2, 3]. There are known different mathematical models that are used in CS to prove image individuality [1, 3, 5–8]. Each of those models focuses on identification accuracy enhancement; however, there is no any other better model free off defects [1, 3].

2. RIDGE COUNT

In dactyloscopy when compare the patterns they usually bear on minutiae and ridge count between them [3, 5]. Such method reflects not only realization of different designation CS but also standards requiring its implementation [9]. However, ridge count, which in Criminalistics measuring along the straight line between two minutiae as discrete random quantity X with integral values, is characterized with instability. The reason of that is stochastic nature of biometric parameter [3], conflicting to existing conception of ridge count measuring. It is evidently, if not to deny heavy noisiness of FI. However, there are many other factors, which are visualized in different operating conditions and not accounted just the noise.

During fingerprint latent formation, minutiae can mutate. At mutation, an ending is closing into the bifurcation, but bifurcation is tearing into ending. This is frequently met first type mutation. [1]. Minutiae type – bifurcation and ending – not always help to consider it even at trivial alteration of a pattern. For example, in figure 2 ridge count value 3 transforms into value 2 at closing of v ending into bifurcation (number of transitions from line to line is summarized). At such type of mutation of ending u ridge count becomes appreciably different.

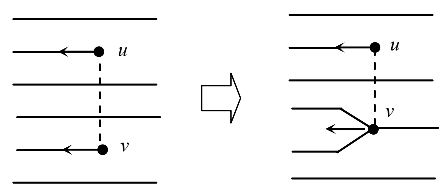

Figure 2. First type of mutation and change of ridge count between endings

Moreover, it is observed dependence of ridge count value x from lines curvature [7, 8]. At FI deformation lines curvature is changing. In figure 3 the value x measured along the doted line is sensitive to deformation, which results in pattern lines straightening.

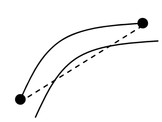

Figure 3. Instability of ridge count value in the area of significant curvature of lines

The other effects of latent print formation are also influence, they deform pattern saving good quality of FI. These are both minutiae displacement along lines, and dead zones appearing near minutiae, if measuring realized close to them, and indeterminacy of minutiae multitude choice, up to which ridge count is measured, and ridge count ambiguity at its changing along the lines [3, 7, 8]. Thus, stochastic nature of biometric parameter should be considered in FI identification procedure, for example, by means of tolerance extension for discrete random quantity X deviation. As result, it is increase of identification errors.

3. NEW RIDGE COUNT MODEL

Ridge count rely on minutiae, multitude of which is in terms of

$$L_m = \{M_i = \{(x_i, y_i), \alpha_i, t_i\} \mid i \in 1..n\}, \qquad (1)$$

where M_i – minutiae with i index number; (x_i, y_i), α_i, t_i – coordinates, direction and type of minutiae; $n = |L_m|$ – cardinal number. Minutiae are detected in informative area of an image, which is darkened in figure 1.

Coordinates (x_i, y_i) are determined as coordinates of skeleton node [2]. Direction α_i is the angle determined with a chain of skeleton nodes for ending and a tree of chains for bifurcation. It points to the area of lines number increasing (look figure 2). Type $t_i \in \{0,1\}$ is determined with skeleton nodes valence like the nodes of graph [4].

3.1 Topological vectors

Minutiae topological vectors L_v model is based on multitude L_m (1) in terms of

$$L_v = \{V_i = \{(e_j, n_j)\} \mid i \in 1..n, j \in J = 1..m\}, \qquad (2)$$

where V_i – minutiae topological vector; n – cardinal number, $n = |L_v| = |L_m|$; i – index as number of topological vector; j – link number; e_j – event, formed with minutiae with number n_j; m – links number at section depth, equal to g lines, as follows

$$m = \begin{cases} 4g+1, & \text{if } t_i \in \{1\} - \text{ending}, \\ 4g+3, & \text{if } t_i \in \{0\} - \text{bifurcation}. \end{cases} \qquad (3)$$

Let us dwell on the procedure of topological vectors synthesis.

In the informative area of FI lines are marked out, skeleton is formed and two types of minutiae are detected: endings and bifurcations (look figure 1). Direction of minutiae M_i points to the area of lines number increase. It is parallel to tangent of papillary line image. Each minutia is numbered. Also for each minutia, we draw projections to the right and to the left transversely to the direction vector of the minutiae onto adjacent lines of skeleton and fix the projections. In figure 4 the projections are shown with dotted lines and two corresponding nodes of skeleton on the lines 1 and 2 are painted over.

Consider the node p of a skeleton, which conform to minutia M_i, and draw the section across its coordinates (x_i, y_i) to the right and to the left at a distance of g lines transversely to the tangent to lines being crossed and enumerate on gyrate the dissected lines (hereinafter 'links'), which turn clockwise. The section passes and traces the lines curvature [6]. The section g depth is varying from one to eight lines to the right and the same to the left. One line in a section forms two links, total number of which is calculated according to (3).

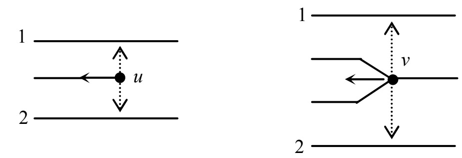

Figure 4. Projections for ending and bifurcation

Topological vector V_i is determined by the section. Then we follow the move of every link by turns on each link, not leaving it beginning from the section until finding another minutia, located on the link, or a minutiae projection, located on an adjacent line to the right or to the left of the link. Possible events are shown in figure 5 and indicated with number in binary code.

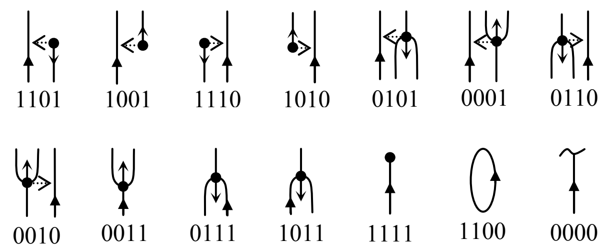

Figure 5. Events

The number of minutia, initiated the event is associated with the event as the number (14 in sum) detected on the link. The event is tied to the link number. For 0000 and 1100 events minutiae numbers are absent: the line either breaks in the uninformative area or closes and there are no either minutia no projection on the line. Event and minutia number form ordered pair (e_j, n_j), corresponding to j-link.

Definition 1. Topological vector V_i is the numbered set of links with ordered pairs (e_j, n_j).

Bits of codes for the event determines minutiae type, its direction and location, against the link direction and others (look figure 5). Topological vector is built for each minutia.

Hereon the topological vector formation is finished.

In figure 6 in section for ending 19 the links are enumerated as 0–16, and corresponding topological vector is shown in the table 1. Such section for bifurcation 19 is shown in figure 7, and topological vector, formed with links 0-16, is shown in the table 2. The sections are shown with doted lines. They cut lines into links, enumerated on gyrate clockwise. These figures show well-known mutation of ending 19 into bifurcation 19.

Comparative analyses of topological vectors with Sparrow method [7] show a series of advantages. *Firstly*, the section is built along the curve, following the direction of lines curvature, not along the straight line. *Secondly*, during event determination minutia projection is taken into account, which in case of mutation avoids the loss of information in the form of event omission during link tracing. *Thirdly*, links are enumerated on gyrate without links omission, which allows increasing the section size keeping the contents of shorten and lengthen vectors common part. This increases stability and informativity of mathematical model.

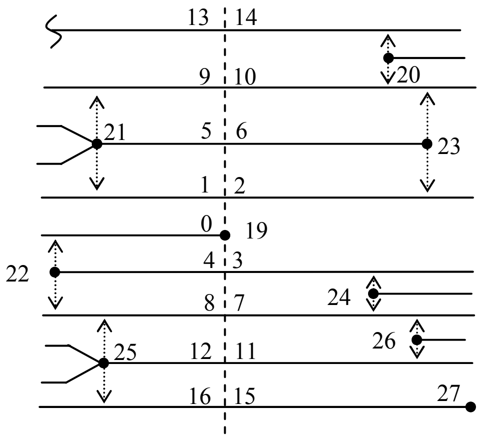

Figure 6. Section for ending

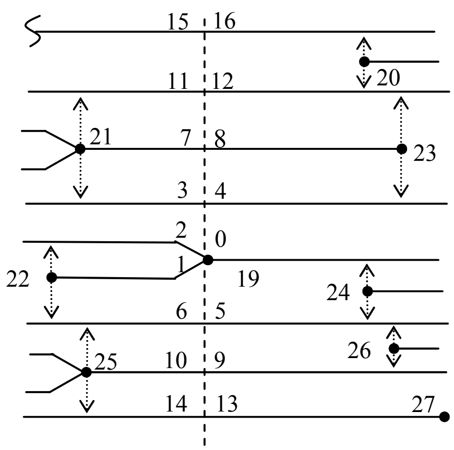

Figure 7. Section for bifurcation

Table 1. Topological vector of ending

Link number	Event	Index
0	1110	22
1	0001	21
2	1110	23
3	1001	24
4	1111	22
5	0011	21
6	1111	23
7	1010	24
8	0010	25
9	0010	21
10	1010	20
11	1010	26
12	0011	25
13	–	–
14	1001	20
15	1111	27
16	0001	25

Table 2. Topological vector of bifurcation

Link number	Event	Index
0	1001	24
1	1111	22
2	1110	22
3	0001	21
4	1110	23
5	1010	24
6	0010	25
7	0011	21
8	1111	23
9	1010	26
10	0011	25
11	0010	21
12	1010	20
13	1111	27
14	0001	25
15	–	–
16	1001	20

3.2 Ridge count vectors

Disadvantages of the classical ridge count are compensated in the model of ridge count based on FI topological vectors. Ridge count model L_r based on model L_v using (2) in the following form

$$L_r = \left\{ R_i = \left\{(r_k, n_k)\right\} \mid i \in 1..n, k \in 1..t \right\}, \tag{4}$$

where R_i – minutiae ridge count vector is the ordered set of ordered pairs (r_k, n_k) with k index; n – cardinal number and $n = |L_r| = |L_v|$; i – index as a number of ridge count vector; k – link number; t – quantity of links, equal to quantity of minutiae in R_i vector; r_k – ridge count value, and n_k – minutiae number, using (1) on k-link.

Let us dwell on the procedure of ridge count vectors building.

On the basis of L_v using (2) auxiliary topological vectors are calculated in terms of

$$L'_v = \left\{ V'_i = \left\{(e_j, n_j, r_j)\right\} \mid i \in 1..n, j \in J = 1..m \right\}, \tag{5}$$

where r_j – ridge count value at j-link and

$$r_j = c_j + f(j, e_j)\ ; \tag{6}$$

c_j – transition count from line to line up to j-link in the section for topological vector V_i using (2); function f determines the correction value depending on j link and e_j event value.

Let s_j – state of the j-link, $s_j = \varphi(e_j)$, $s_j \in \{0,1\}$. Here $s_j = 0$ if the direction vector of minutiae directed along the link course; $s_j = 1$ if the direction vector of minutiae directed contrariwise to the link course. Function φ is realized in terms of

$$\varphi(v_i) = \{\_, 0, 0, 0, \_, 1, 1, 1, \_, 0, 0, 1, \_, 1, 1, 1\}\ ,$$

where $\{\_\}$ – skip code. We can see that $\varphi(1)=0$ or $\varphi(5)=1$. So k-minutiae in vector V_i' linked with $s_j=\varphi(e_j)$ can be associated with state of the k-minutiae s_k.

Let the next in order k-minutiae in vector V_i' is met for N times and its number is $h=n_j$. Then it is possible to calculate an average value of ridge as follows

$$r_k=\frac{1}{N}\sum_{\substack{j\in J\\ n_j=h}} r_j\,, \qquad (7)$$

corresponding to the minutia M_{n_j} in vector V_i'. The quantity t of such average values equal to the quantity of minutiae in vector V_i'. Ridge count vector is calculated as follows

$$R_i=\{(s_k,r_k,n_k)\},\ i\in 1..n,\ k\in 1..t\,,$$

where for every k-minutiae there is ordered triplet (s_k,r_k,n_k), where n_k – minutiae number, for which it is calculated the following r_k; s_k – state of the k-minutiae; $n_k=n_j$. Quantity of t links using (4) not exceeds the quantity of m links using (3).

Definition 2. Ridge count vector R_i is the numbered set of links with ordered triplets (s_k,r_k,n_k).

Links can be ordered by an angle in polar coordinate system, oriented along the direction of minutiae M_i, for which auxiliary vector V_i' is determined. In this case, an order of minutiae enumeration is similar to [5], but quantity of them is substantially less.

3.3 Calculation of correction function

Auxiliary topological vector from (5) allows determination of the value r_j of ridge count. Let us determine the set $L=\{1110,1010,0110,0010\}$ of «left» projections and set $R=\{1101,1001,0101,0001\}$ of «right» projections in the form of enumeration of events. Let us assign a set O_t of links of basic line for the end $O_1=\{0\}$ and bifurcation $O_0=\{0,1,2\}$, where $t\in\{0,1\}$ – minutiae type from (1, 3). Let us make four sets of links Q_1,Q_2,Q_3 и Q_4 for the quadrants in the coordinate system, oriented along the direction of minutiae M_i. This is curvilinear coordinates; ordinate axis of which is curving along the section of topological vector, and an abscissa – along the basic line. These sets of links are shown in the table 3 for the ending in figure 6 and bifurcation in figure 7.

Table 3. Links in quadrants

Ending	Bifurcation
$Q_1=\{4,8,12,16\}$	$Q_1=\{6,10,14\}$
$Q_2=\{3,7,11,15\}$	$Q_2=\{5,9,13\}$
$Q_3=\{2,6,10,14\}$	$Q_3=\{4,8,12,16\}$
$Q_4=\{1,5,9,13\}$	$Q_4=\{3,7,11,15\}$

Relying on the following sets L,R,Q_t,Q_1,Q_2,Q_3 и Q_4, function of ridge count value correction can be written down in terms of

$$f(j,e_j)=0{,}5\ sign(j,e_j)\,, \qquad (8)$$

where

$$sign(j,e_j)=\begin{cases}+1, & \text{if } e_j\in L\wedge j\in Q_1,\\ -1, & \text{if } e_j\in R\wedge j\in Q_1,\\ -1, & \text{if } e_j\in L\wedge j\in Q_2,\\ +1, & \text{if } e_j\in R\wedge j\in Q_2,\\ +1, & \text{if } e_j\in L\wedge j\in Q_3,\\ -1, & \text{if } e_j\in R\wedge j\in Q_3,\\ -1, & \text{if } e_j\in L\wedge j\in Q_4,\\ +1, & \text{if } e_j\in R\wedge j\in Q_4,\\ +1, & \text{if } j\in Q_t.\end{cases} \qquad (9)$$

Correction function from (8) for every projection on link adds or subtracts a half of transition count from one line to another line in a section for auxiliary topological vector using (5). Correction sign from (9) is determined with projection direction. If an event e_j corresponds to projection, which approaches the minutiae M_{n_j} to the basic line with links from O_t, then correction sign is negative, otherwise – positive. Intuitively it seems to be understood. However, the result of calculation using (7) is equal to fractional values. Actually, correction function and average values calculation help to convert ridge count values into fractional numbers. Under mutation, ridge count modification can remain the same as in classic approach, or become less. This positive affect is shown in the table 4 and table 5, where are shown the results of ridge count calculation for ending 19 in figure 6 and bifurcation 19 in figure 7. Minutiae in tables are ordered by angle in polar coordinate system, which corresponds to Shmakov's approach [5]. We can see from tables that for minutiae of the same name changing of ridge count Δr in case of mutation of first kind not exceed a unity. Values Δr calculation is shown in the table 6.

If minutiae located on links are subject to mutations, then ridge count change, less than a unity. It is being met more often. For example, if bifurcation 21 in figure 6 will tear to ending between 5-th and 9-th links, and in figure 7 will remain, then $\Delta r=0{,}5<1$; for classic ridge count $\Delta r=1$.

Table 4. Ending ridge count vector

Number of link	State	Ridge count	Index
0	1	0,75	22
1	0	3	25
2	1	4	27
3	0	2,5	26
4	0	1,5	24
5	1	1,75	23
6	0	3,5	20
7	0	2	21

Table 5. Bifurcation ridge count vector

Number of link	State	Ridge count	Index
0	1	0,(3)	22
1	0	2	25
2	1	3	27
3	0	1,5	26
4	0	0,5	24
5	1	1,75	23
6	0	3,5	20
7	0	2	21

Table 6. Ridge count change in vector

Ridge count change	Index
0,41(6)	22
1	25
1	27
1	26
1	24
0	23
0	20
0	21

Let us indicate advantages of ridge count vector. *Firstly*, at minutiae mutation, on both basic line and links, fractional ridge count allows reducing the changes of its values. *Secondly*, ridge count values do not depend on lines curvature, as if the procedure of ridge counts measuring along the straight line to be excluded. *Thirdly*, ridge count values are measured along the lines the same stably as across them. *Fourthly*, minutiae set, which are the part of ridge count vector, is unambiguously determined and limited with topological structure of pattern, but not conditional radius-vectors and the same mechanisms. *Fifthly*, state of minutiae is steady to line curvature in the area of loops, deltas and wholes. *Sixthly*, ridge count vector is steady to image deformation, which distorts the line curvature, as since it inherit positive features of topological vector. This increases the stability and informativity of ridge count vectors.

More over, ridge count vector is invariant to scale and rotating of FI the same as classical ridge count.

4. CONCLUSION

In the current work, the mathematical model of FI based on ridge count vectors and topological vectors is suggested. In spite of different missions in construction of either of the two models, it is indicated the total feature of them: the models use topological description of FI. As a result, such models describe image more completely as compared with models, suggested earlier [3, 5, 7, 8]. For the first time ridge count value means fractional number.

Requital for mentioned advantages is heightened complexity of models calculation. However, the models contain mechanisms that compensate influence of minutiae mutation, image deformation and lines curvature. This increases the stability of FI template.

Further direction of research work is seen in synthesis of mathematic models, oriented on FI presentation in the form of lines ridge count vectors, but not minutiae, with fractional ridge count. Such researches will help in realization of FI indexation mechanisms, which multiply accelerate procedures of fingerprint images identification.

5. REFERENCES

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