# Topological Mapping Complex 3D Environments Using Occupancy Octrees

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## Abstract

Global path planning is a challenging problem arisen in many fields of research. Unfortunately, path planning algorithms have relatively high complexity that extremely grows with the input data volume. Being oriented on accurate metric schemes, traditional local path planning methods have significant limitations in the case of large-scale environments. Their inability to use overall information on the whole environment creates critical shortcoming in global planning. Topological schemes try to overcome this drawback by representing the original environment by means of route graphs. Topological schemes scale better than metric ones, but being resistant to geometric representation errors may yield incorrect or suboptimal solutions.

In the paper we propose an effective method of generating topological maps for global path planning in complex large-scale 3D environments. The method utilizes adaptive spatial decomposition in conformity to occupancy octree structures and uses original criteria for identifying spaces and gates. Thus generated maps provide for whole coverage of environments and enable to effectively resolve multiple path planning requests using graph search algorithms. Conducted experiments proved the feasibility and effectiveness of the method presented as well as its suitability to industry meaningful problem statements.

***Keywords:*** *Global path planning, Topological and metric schemes, Collision Detection.*

## 1. INTRODUCTION

Global path planning is a challenging problem arisen in many fields of research, including global positioning systems (GPS), autonomous robot navigation and very large-scale integration design (VLSI). The problem is of particular interest to construction planning community facing the requirements of trustworthiness and feasibility of project schedules [10]. Correct schedules must avoid any conflicting situations at project sites and assure the existence of collision-free paths for installed construction elements and deployed equipment. Ultimately it would enable detecting and anticipating problems at earlier planning phases and reducing risks and waste at the final phases.

Unfortunately, global path planning algorithms have relatively high complexity that extremely grows with the input data volume. Well-known metric schemes like configuration spaces, generalized cones, voronoi diagrams, visibility graphs, cell decompositions correspond directly to original geometric representation of the explored environment and provide implicit information about the traversability relations among different places. Being based on metric schemes, popular path planning methods such as probabilistic roadmaps (PRM), rapidly exploring random trees (RRT), and potential fields succeed on local statements. However, their inability to use overall a priory information on the whole environment creates critical shortcomings in global planning.

Topological schemes try to overcome these drawbacks by representing the original environment by means of route graphs. Typically, vertices of such graphs are associated with identifiable locations and edges — with possible routes between them. Topological schemes scale better than metric ones, but being resistant to geometric representation errors may yield incorrect or suboptimal solutions [7]

Integration of metric and topological schemes looks most promising approach to leverage advantages of both paradigms. It can be performed by extracting a topological map from metric representations and by annotating topological elements with metric information. Occupancy grids are usually chosen as a metric representation which is easy to calculate and to update. To reduce their volume and to obtain a reasonable partitioning, the grid cells are grouped in homogeneous regions. Each such region becomes a vertex of the generated topological map and if there is a feasible path joining two regions then their vertices are connected via corresponding edge.

Extensive research efforts have been directed toward topological mapping problems. This partitioning can be performed by using quadtrees [13]. However, the optimality of the resulting partition depends strongly on the distribution of the obstacles in the environment [2]. Sometimes obstacle boundaries are modeled by means of straight lines [1]. The main disadvantage of this method is that it can not deal correctly with irregularly shaped regions nor with walls which are not parallel or orthogonal. The topological map can be extracted from the grid by analyzing the shape of free space by means of the mathematical morphology image processing tool [4]. However, only explored regions can be represented at topological level. Region shape criteria were proposed in [12] to split free space into homogeneous regions. It is reported in [6] that these maps, as well as other self organizing maps like the colored map and the growing neural gas, produce unintuitive tessellations of free space. Wall histograms were proposed in [6] to resolve these problems. However, this method provides no suitable topologies for navigating in free space. Many researchers tried to obtain adequate topological map by identifying features like virtual doors, narrow passages, corners, middle lines [3], [9]. Although their methods work well for particular domestic cases, scarcely that they match to complex 3D environments, particularly, indoor and outdoor environments simulating real construction project sites.

In the paper we propose an effective method of generating topological maps for complex indoor/outdoor environments. The method utilizes adaptive spatial decomposition in conformity to occupancy octree structures and uses original criteria for identifying spaces and gates. Thus generated maps provide for whole coverage of environments and enable to effectively resolve multiple path planning requests using graph search algorithms.

The rest of the paper is organized as follows. Section 2 describes peculiarities of adaptive octree structures utilized as a metric occupancy representation. In Section 3 we present an original algorithm and criteria for extracting a topological map from the metric representation. The algorithms to find suboptimal routes in the topological graph and some results of conducted experiments are mentioned in Section 4. In conclusions we summarize benefits of the topological mapping method proposed.

## 2. OCCUPANCY OCTREE

To simplify the discussed motion planning problem and to avoid computationally expensive analysis of whole 3D environments, so-called spatial decomposition is usually applied. It assumes subdividing the environment space into cells and determining occupancy status for each localized cell. Using the occupancy metric scheme, path planning problem can be solved more efficiently by successive finding neighboring free cells and navigating over them from a starting point of the moved object to its destination point. Importantly, it can be done under very general assumptions about the simulated environment.

We suggest the environment is composed of objects which may be geometric primitives, algebraic implicit and parametric surfaces like quadrics, NURBS and Bezier patches, convex and non-convex polyhedrons, solid bodies given by constructive solid geometry or boundary representation. No matter which geometric models are adopted. It is assumed only that there is a common function for the interference identification between any environmental object and any given box. The testing result is an occupancy status taking the values 'grey', 'black' or 'white' and pointing whether the box is partially occupied, entirely full, or entirely empty correspondingly.

In this paper we confine ourselves to the case of static environments, although the method presented can be applied to dynamic and pseudo-dynamic environments too. As opposed to many works addressing to path planning, no specific restrictions are imposed upon the geometric representation of both simulated environment and moved objects considered as its intrinsic parts.

Adaptive cell decomposition is used to reduce the number of cells suffered to the analysis and to waste less memory storage space and computation time. Contrary to regular cell decomposition, it is a good tactic for the complex indoor/outdoor environments simulating construction sites and containing large regions with the same traversability. At the same time adaptive cell decomposition imposes problems for dynamic and pseudo-dynamic environments. One good solution is the use of framed trees as suggested in [11]. However, in high clutter environments framed structures can be less efficient than regular grids due to the overhead required to keep track of the cell sizes and locations.

To generate the occupancy octree the well-known adaptive decomposition technique is applied. It begins by imposing a large size cell over the entire planning space. If a grid cell is partially occupied, it is sub-divided into eight equal subparts or octants, which are then reapplied to the planning space. These octants are then recursively subdivided again and again until each of the cells is either entirely full or entirely empty. The subdivision process is interrupted also for refined cells if their size becomes equal or smaller than a given tolerance of the generated metric representation. The resulting octree has grid cells of varying size and concentration, but the cell boundaries coincide very closely with the obstacle boundaries. An example of the octree representation is shown in Figure 1b. It was generated for a simple building model presented in Figure 1a.

The cells of the deployed octree are then marked as 'grey', 'black' or 'white' depending on their occupancy status that points out whether the cell is partially occupied, entirely full, or entirely empty. Note that sometimes it is difficult to identify entirely full cells for the environments consisted of, so-called, polygon soups rather than solid primitives or assemblies. In such situations the subdivision process has to be recursively continued under suggestion that the cells are partially occupied regions.

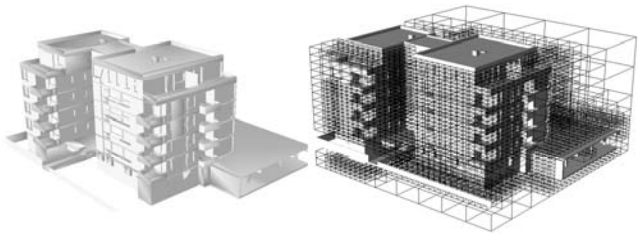

**Figure 1:** (a) Geometric representation of the building, (b) Deployed occupancy octree.

To make the metric representation more constructive for spatial reasoning and topology extracting, it is proposed to enrich the octree structure by Euclidian distance field values. For this purpose empty cells store distances to the nearby obstacles. To simplify computations, the distance estimates can be obtained using already deployed metric scheme and avoiding consumable analysis of the original geometric representation. In the proposed algorithm the distances are computed from the center of each empty cell to the nearest faces, edges or corners of the neighboring cells (entirely or partially) occupied by the environment objects. Neighboring cells are determined in a way similar to the backtracking method [5] by finding common ancestor and by descending from it to adjacent cells of the explored octant.

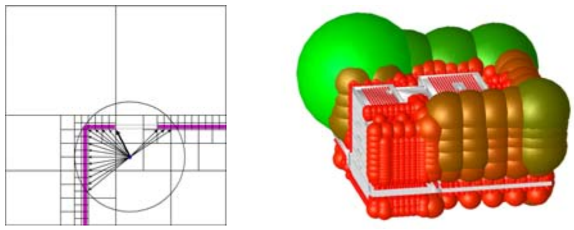

**Figure 2:** (a) Estimated search area and neighboring occupied cell, (b) Distance field distribution.

A principal distinction of the presented algorithm is that it collects also indirect neighbors which are located not far than search radius $r$ from the explored octant. An initial estimation for the search radius is given by the expression $\sqrt{3}(\frac{3}{2}a - d)$, where $a$ is the linear size of the explored octant and $d$ is the size of the smallest cell coincident with the given metric tolerance. Indeed, by construction the parent of the explored empty octant may be only a grey cell and it must contain at least one occupied child cell. Evidently that it is located not far than $\sqrt{3}\,(a-d)$ from the center of the parent cell and not far than the given value from the center of the explored octant. As new neighbors are found, the estimated search radius can be refined and replaced by the current value of the distance. This leads to additional savings on traversing and processing cells. An estimated search area and an occupied cell nearby to the central empty octant are shown in Figure 2a as a circle and a bold arrow correspondingly. Figure 2b illustrates the distance field distribution computed for the building model and the occupancy octree presented above.

## 3. EXTRACTING TOPOLOGICAL MAP

As explained, the occupancy octree is an useful metric representation to perform path planning in 3D environments by successive navigating over empty octants and avoiding the environment obstacles. However, being represented by huge number of cells the octree prevents efficient coverage in large-scale environments. This observation made us to follow “space-gate” reasoning paradigm and to support corresponding topological scheme. Both spaces and gates are considered as non-overlapping, simply connected subsets of empty cells of the occupancy octree. Principal difference between introduced categories is that the spaces approximate large free regions of the environment, whereas the gates — small narrow regions.

Therefore, the objective of this study is to extract a topological scheme from available metric information. This problem attracted many researches which tried to obtain adequate topological scheme by identifying features like virtual doors, narrow passages, corners, middle lines [9]. Although their methods work well for particular domestic cases, scarcely that they match to complex environments simulating real construction projects. The proposed algorithm for extracting a “space-gate” topological scheme looks very promising from this point of view.

Underlying principle for detecting free regions is to find the cells where the distance field reaches local maxima. Each such maximum originates a subset of empty, simply connected cells surrounding it and having distance values not exceeding the local maximum. Opposite to centerline algorithms oriented on rectangular occupancy grids and steepest descent by gradient vector approximations, our algorithm is applicable to arbitrary irregular grids and octrees which of special value for the discussed global path planning problems.

To explain how the algorithm works, let us define binary relations of the adjacency, dominancy and origination among empty cells of the occupancy octree $C$. The cells $c', c'' \in C$ are adjacent (or $c' \sim c''$) if and only if there is a common face belonging to boundaries of both cells. The cell $c' \in C$ dominates over the cell $c'' \in C$ (or $c' > c''$) if and only if the cells are adjacent and estimated distance value for the cell $c'$ is larger than the corresponding value for the cell $c''$. And, finally, the subset $C' \subseteq C$ is originated from the cell $c' \in C'$ ( $c' \triangleright C'$ ) if and only if for any $c'' \in C'$ there is a sequence $c_1, c_2, ..., c_n \in C'$ so that $c' > c_1, c_1 > c_2, ..., c_n > c''$ and there is no dominating cell $c''' \in C'$ for $c'$ so that $c''' > c'$. The introduced adjacency relation is symmetric, reflexive and transitive. The dominancy relation is transitive.

We define the regions to be subsets of cells $R_1, R_2, R_3, ... \subseteq C$ obtained by transitive closure of the dominancy relation on a set of all empty cells of the octree $C$. Then the spaces $S_1, S_2, S_3, ... \subseteq C$ can be defined as non-intersecting subsets of regions $S_1 = R_1 \setminus \{R_2 \cup R_3 \cup ...\}$, $S_2 = R_2 \setminus \{R_1 \cup R_3 \cup ...\}$, $S_3 = R_3 \setminus \{R_1 \cup R_2 \cup ...\}$. Consider now a remained subset $G = \{R_1 \cap R_2\} \cup \{R_1 \cap R_3\} \cup \{R_2 \cap R_3\} ... \subseteq C$ representing all the intersected regions with more than one originating cell. Cells of the subset $G$ with the same combination of originating cells are grouped to form the gates $G_1, G_2, G_3, ... \subseteq G \subset C$. One would assume that the extracted spaces and gates have no mutual intersections $C_i \cap C_j = \varnothing$, $G_k \cap G_l = \varnothing$, $C_i \cap G_k = \varnothing$, $i \neq j$, $k \neq l$ and they form an exact cover of the original set $S_1 \cup S_2 \cup ... \cup G_1 \cup G_2 \cup ... = C$.

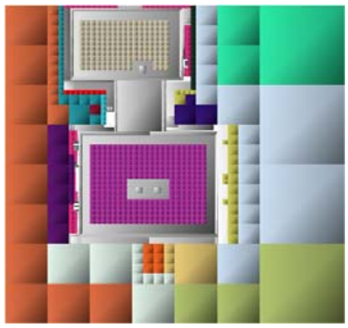

**Figure 3:** Space and gate regions identified in the occupancy octree.

The introduced definitions and obtained formula give a constructive algorithm to identify spaces and gates and to extract a topological map from the metric representation. An important advantage of the algorithm is an avoidance of any domestic feature analysis and its applicability to both indoor and outdoor environments. An example of the space and gate identification is shown in Figure 3.

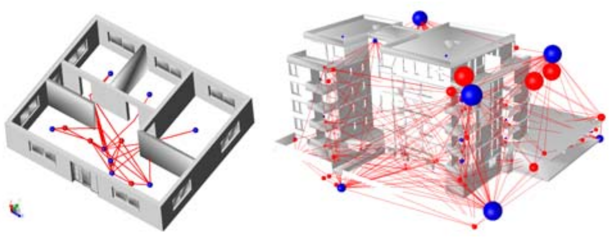

**Figure 4:** A bipartite “space-gate” topological graph for indoor and outdoor environments.

All the recognized spaces and gates match to corresponding vertices of the generated bipartite topological graph. Space vertices are connected by edges with incident gate vertices, but not with other space vertices. Similarly, gate vertices are connected with incident space vertices, rather than with other gate vertices. Thus, spaces and gates are alternated when navigating over environment and traversing the topological graph. A bipartite topological graph generated for indoor and outdoor environments is shown in Figure 4.

## 4. SOME EXPERIMENTAL RESULTS

Using the "space-gate" topological map and graph search algorithms like classical Dijkstra's and Tarjan's algorithms, we can determine what the possible path to get to a certain destination is. The spatial path can be easy formed from waypoints lying in the centers of incident spaces and gates associated with the found route. Then the path is checked against potential collisions and, if necessary, it is corrected using known modification of the RRT algorithm that assumes growing trees from neighboring waypoints in opposite directions. For details see randomized kinodynamic planning [8]. Figure 5 provides an example of the resulting path in the presented building model obtained using both global and local planning strategies.

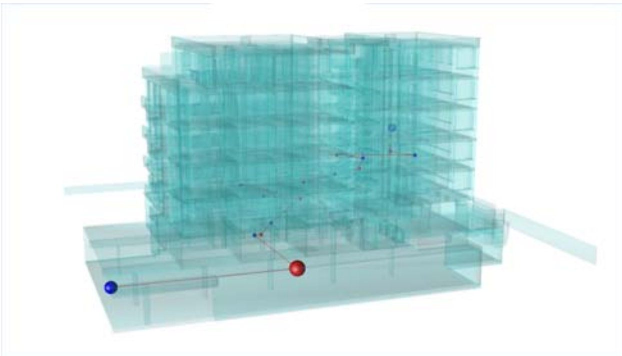

**Figure 5:** A resulting path obtained using both global and local strategies.

To validate the topological mapping method proposed and to estimate its practical benefits, we have implemented a program and conducted timing experiments, in which a middle-size building model was applied. Although the model was strongly structured in accordance with IFC standard, in fact a polygon soup consisted from about 500,000 triangles was taken as original 3D geometry data. A series of the experiments corresponded to different tolerance values of the metric representation limited by the regular grid sizes.

The experiments showed that the method demonstrates polynomial complexity grow avoiding any exponential explosion. For the grid 52 x 56 x 30 the analysis took totally 107 CPU seconds on a typical computer configuration Core 2 Duo E8600 processor (2.13 GHz), 2GB of RAM (800 MHz). After the topological map has been constructed, search in the graph and final validation of the found spatial routes took no more than a few seconds that looks like quite promising result giving an opportunity to interactively resolve path planning problems.

## 5. CONCLUSION

Thus, the effective method of generating topological maps for large-scale 3D environments has been proposed. The method utilizes occupancy octree structures and uses original criteria for identifying spaces and gates. Thus generated maps provide for whole coverage of environments and enable to effectively resolve multiple path planning requests using well-known graph search algorithms. Conducted experiments proved the feasibility and effectiveness of the method presented as well as its suitability to global path planning problems in complex indoor-outdoor environments.

## 6. REFERENCES


[1] Alero A., Milln J.R and Floreano D. *(1999) "Efficient learning of variable-resolution cognitive maps for autonomous indoor navigation." IEEE Trans. on Robotics and Automation, 15(6), 990-1000*

[2] Chen D.Z., Szczerba R.J. and Uhran J.J. *(1997) "A framed-quadtree approach for determining euclidean shortest paths in 2D environment" IEEE Trans. on Robotics and Automation, 13(5), 668-680.*

[3] Choi J. W., Choi M. Y. and Chung W. K. *(2009) "Incremental Topological Modeling using Sonar Gridmap in Home Environment." In International Conference on Intelligent Robots and Systems, 3582-3587.*

[4] Fabrizi E. and Saffiotti A. *(2000) "Extracting Topology-Based Maps from Gridmaps." In International Conference on Robotics and Automation, 2972-2978.*

[5] Kim J., Lee S. *(2009) "Fast neighbor cells finding method for multiple octree representation" In IEEE Int. Symposium on Computational Intelligence in Robotics and Automation. 540-545.*

[6] Kraetzschmar G., Sabaltong S., Enderle S. and Palm G. (2000) *"Application of neurosymbolic integration for environment modeling in mobile robots", in Wermter and Sun (eds) Hybrid Neural Systems, Springer, Berlin.*

[7] Lamarche F. *(2009) "TopoPlan: a topological path planner for real time human navigation under floor and ceiling constraints." Computer Graphics Forum. 28 (2). 649-658.*

[8] Lavalle S. M. *(2006) "Planning algorithms." Cambridge University Press, Cambridge, UK.*

[9] Myung H., Jeon H. M., Jeong W. Y. and Bang S. W. *(2009) "Virtual Door-Based Coverage Path Planning for Mobile Robot." In Federation of International Robot-soccer Association, 197-207.*

[10] Semenov V.A., Kazakov K.A., Morozov S.V., Tarlapan O.A., Zolotov V.A., Dengenis T. *(2010) "4D modeling of large industrial projects using spatio-temporal decomposition." In eWork and eBusiness in Architecture, Engineering and Construction, eds. K. Menzel and R. Scherer, 89-95.*

[11] Szczerba R.J., Chen D.Z., Uhran J.J. *(1998) "Planning Shortest Paths among 2D and 3D Weighted Regions Using Framed-Subspaces" International Journal of Robotics Research. 17(5). 531-546.*

[12] Thrun S., Bucken A., Bulgard W., Fox D., Frohlinghaus T., Hennig D., Hofmann T., Krell M. and Schmidt T. (1998) *"Map learning and high-speed navigation in RHINO" MIT/AAAI Press, Cambrige-MA.*

[13] Zelinsky A. *(1992) "A mobile robot navigation exploration algorithm", IEEE Trans. on Robotics and Automation, 8, 707-717.*