Image segmentation by means of optimal approximations

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Abstract

In the report the segmentation problem of a priory unknown image is discussed. The solution is developed in framework of Mumford–Shah model. To formalize the result of segmentation of any image into each number of segments avoiding an appeal to prescribed image segmentation method or algorithm the optimal image approximations are used. The insufficiency of iterative segment merging employed in Mumford–Shah model to obtain the optimal approximations of an image is established. The improvement of optimization results is demonstrated.

Keywords: *Mumford–Shah Model, Optimal Approximation, Segment Number, Standard Deviation.*

1. INTRODUCTION

Image segmentation is an important problem in preliminary image analysis described as certain partitioning of an image into proper segments. The one of known correct solutions consists in specifying a certain functional, which attains the minimal value for the segmented image compared with the values of this functional for arbitrarily partitioning of an image into segments. At that the definition of image segmentation result does not appeal nor to a visual perception nor to a priori prescribed image segmentation technique. So for image of any content it appears possible to obtain the minimal values of the numerical functional and calculate the proper segmented representations. But to do so some known theoretical and computational obstacles should to be overcome. We start to solve the mentioned task in terms of optimal piecewise constant image approximations which are used in the Mumford–Shah model [2, 4, 6, 7, 9–12].

According to the Mumford–Shah model, a functional to be minimized usually depends on standard deviation of approximation from an image and the total length of boundaries between image segments. Accounting for segment boundaries was introduced to describe a non-trivial minimum of the functional for a certain number of segments. At the same time it complicates the optimization problem and forces the use of an extra control parameter [6, 7, 9, 10]. In special case of ignoring of segment borders the optimization reduces to minimizing the standard deviation of approximation from an image [2]. In this case the optimal approximation appears trivial and coincides with the partition of an image into separate pixels, or segments of identical pixels corresponded to zero value of the functional. However, for meaningful statement of the problem avoiding any functional complications it seems perfect to minimize the functional for each possible segment number that guarantees the existence of solutions and simplifies the interpretation of the functional. On the first sight the requirement of minimizing of functional for each segment seems a generalization of the problem statement that commonly used. But it should be noted that the consideration of a sequence of optimal approximations instead of single approximation maintains the logic of practical computations as the selection of the desired image approximation from a set of calculated ones is often provided within a supplementary «stopping condition» [2, 6, 11] that treated in addition to initial minimization requirement. Nevertheless, for more effective utilization of the model it is useful to clarify and carefully experimentally verify the concept of optimal approximation avoiding an overestimation of the results of computer calculations.

2. OPTIMAL APPROXIMATION PROBLEM

Let us illustrate the notion of optimal approximation by the example of the digital image of the four pixels, comparing the image and its approximation by the standard deviation $StdDev$ of approximation from the image (Fig. 1).

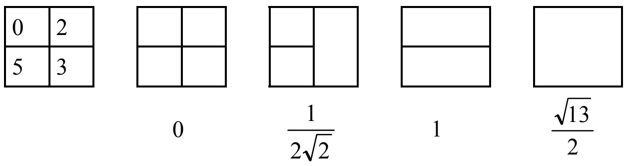

Figure 1: Image partitions inducing optimal approximations.

Leftmost matrix in Fig. 1 is an image example. Intensity values of pixels are written in the cells. Other matrices describe the partitions of the image that are converted to piecewise constant approximations by filling the segments with averaged intensity values. The values of $StdDev$ are written under the matrices.

The piecewise constant image approximation is called *optimal* if in comparison with any other approximation constituted of the same number of segments less differs from the image by the value of the standard deviation $StdDev$, calculated as the square root of the additive square error SE divided by the number of pixels N: $StdDev = \sqrt{SE/N}$.

By direct enumeration of possibilities one can check up that Fig. 1 demonstrates just partitions inducing the optimal approximations. Comparing in Fig. 1 the partitions of the image into two and three segments, it is easy to notice that segments from different partitions are not nested within each other but overlap one another. Therefore, it is impossible to obtain the sequence of optimal approximations of an image into $n = 1, 2, 3, ...N$ segments by iterative segment merging, which generally gives inaccurate results since the second iteration.

It should be noted that the computational problem of the optimal approximation has no an exact practical solution so far, even for individual standard images of actual dimensions. Therefore the reliable computation of non optimal, but only optimized image approximations is further discussed (Fig. 2).

31.60

Figure 2: Nearly optimal piecewise constant image approximation with two segments. The original image is shown on the left and its contrasted approximation is shown on the right.

Fig. 2 by the example of the standard Lenna image demonstrates an optimized approximation which contains two segments. To increase a visual clarity, the averaged within each segment intensity values of pixels in piecewise constant image approximation replaced with the values 0 and 255. The optimized approximation called a *nearly optimal* is characterized by a minimum value of standard deviation $StdDev$ compared with other piecewise constant approximations that obtained in available algorithms. The appropriate value of standard deviation $StdDev$ is written under the approximation.

In Fig. 2 either segment of nearly optimal approximation covers several objects connected to each other with connective lines and other ties. Connectivity in this case means that to recolor the approximation it is enough to click once on the white and black pixels in "Flood Fill" mode of any image editor. To notice the mutual connectivity of the black or white objects, one should pay attention to the partially black and partially white frame around the perimeter of the image approximation. Ignoring the increase of the $StdDev$ caused by the connective elements, it may be derived that the lower limit of the minimum possible standard deviation exceeds the value 30.65. So we suppose that the value of the standard deviation $StdDev = 31.60$ is small enough to nearly optimal approximation possesses the features of the optimal approximation. Most likely, the latter merely presents a slightly larger number of objects. Thus, the nearly optimal approximation of the image prompts a very simple idea of constitution of optimal approximation that presents the object in the image as mutually connected regions.

3. MUMFORD–SHAH SEGMENTATION

The original approach to the problem of image segmentation as a problem of optimization was founded by D. Mumford and J. Shah in former century in [9, 10] followed by a number of subsequent key studies [2, 4, 6, 11, 12] that are implemented into practice of image processing e.g. in program package «ENVI» (Environment for Visualizing Images, http://www.ittvis.com/portals/0/pdfs/envi/Feature_Extraction_Module.pdf). These studies base on the formalization of segmentation problem as the minimization of a functional presented as:

$$SE + \lambda L = \min , \tag{1}$$

where SE (abbreviated «square error») is the sum of squared deviations of intensities within the segment 1 and 2 from intensity mean values, L is the total length of the boundaries between adjacent segments and λ is a parameter which may depend on the number of segments according to version [6] and also on the image itself according to the version FLSA (Full λ–Scheduled Algorithm) [4, 11, 12]. Though the dependence of the lambda from an image violates the clear interpretation of functional, which is divided into two terms, just this version of the Mumford–Shah model is preferably implemented to the practice of image processing due to exclusion of control parameters. In the special case of $\lambda = 0$ the task is reduced to minimization of the standard deviation $StdDev$ of the image from its piecewise constant approximation [2].

In accordance with Mumford–Shah model the segmentation task is solved using the algorithms of iterative segment merging. At that the criterion for merging of adjacent segments 1 and 2 according to FLSA version is presented as:

$$\frac{\Delta SE(1,2)}{l(1,2)} \equiv \frac{(\Delta I(1,2))^2}{l(1,2)} \cdot \frac{S(1)S(2)}{S(1)+S(2)} = \min , \tag{2}$$

where $\Delta I(1,2)$ is the difference of intensities averaged within the segments, $S(1)$ and $S(2)$ are the segment squares (pixel numbers), $l(1,2)$ is the length of common boundary between the segments 1 and 2, $\Delta SE(1,2) \equiv SE(1 \cup 2) - SE(1) - SE(2)$ is the value of nonadditivity of SE, estimated for segments 1, 2 and their union $1 \cup 2$.

In the version [6] of Mumford–Shah model the merging criterion is expressed as:

$$\Delta SE(1,2) - \lambda \cdot l(1,2) = \min \tag{3}$$

and in the special case [2] for zero λ is reduced to:

$$\Delta SE(1,2) = \min \tag{4}$$

The latter is called in [12] «weighted intensity difference».

The formulae (3) and (4) simply describe the increment of the functional value for current iteration of segment merging and provide the strict minimization of (1) at merging of first pair of pixels. To avoid the control parameters in [11] the estimation of λ as the minimal value that formally equates to zero the expression (3) whence formula (2) follows was suggested detrimentally to the source idea of minimizing of functional (1). Perhaps that is why in mentioned papers with exception of [7] there is a shortage of the experimental data that would clearly indicate the obtained values of minimized functional.

In any case the optimized image approximations do not perfectly coincide with the optimal ones, due to the fact that the sequence of the partitions inducing the optimal approximations, in general, is not hierarchical. Moreover the results of the iterative segmentation of an image into several segments depend on the variable intensity of image pixels, the initial partition of the image into indivisible *superpixels*, and even from rounding errors. Therefore, a quantitative assessment of the optimality of the discussed approximations of the image seems to be useful for providing of robust segmentation by selecting the best approximation of available ones. As a unified criterion of approximation optimality it is convenient to accept the standard deviation *StdDev* of the image from piecewise constant approximation.

4. STANDARD DEVIATION ANALYSIS

In Fig. 3 the dependencies of standard deviation *StdDev* on the segment number in the range from 1 to 1000 are presented for standard Lenna image.

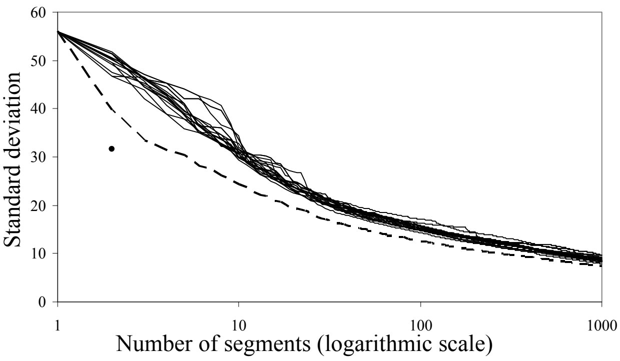

Figure 3: The dependencies of *StdDev* on segment number.

In Fig. 3 the twenty black interlaced curves show the dependencies of *StdDev* on the segment number for different versions of Mumford–Shah model. These weakly decreasing curves describe the hierarchies of nested partitions obtained by iterative merging of adjacent segments with the minimal «weighted intensity difference» (4), minimal value of criterion (2) and a number of experimentally chosen another functions of segment features. To generate the partitions inducing the appropriate piecewise constant image approximations the variation of initial image partitioning into superpixels and normalization of intensities to invariant values with respect to certain image transformations were also used.

Dashed curve corresponds to a non–hierarchical sequence of partitions with overlapping of segments from different partitions. This curve describes the partitions inducing piecewise constant image approximations obtained by iterative merging of adjacent segments with the minimal value of criterion (4) in turn with automatic correction of the segment boundaries accompanied with reduction of the standard deviation value by means of a pilot version of algorithm implementing the methodology of active contours [1, 3, 5, 8] combined with Monte Carlo method [7]. Then the selection of soundness segmentation results according to the monotony condition:

$$n_1 \le n_2 \quad \Rightarrow \quad StdDev(n_1) \ge StdDev(n_2) \qquad (5)$$

was performed, where n_1, n_2 are the numbers of segments in partitions.

The bold dot in Fig. 4 marks the partition inducing the nearly optimal approximation of the image with two segments presented above in Fig. 2. To obtain this partition the segment boundaries were corrected in interactive mode as in [1].

The twenty curves in Fig. 3 fill some strip over the infimum of possible values of the standard deviation for successive values of the segment number. Dashed curve describing a sequence of overlapping partitions, lies lower than twenty curves describing the hierarchical partitions, and for images partitioning into relatively small number of segments shows a noticeable decrease in the standard deviation. It is remarkable that the value of the standard deviation for nearly optimal approximation marked with the bold dot turns out to be one third less than the value for the approximation calculated in Mumford–Shah model by segment merging only. The discussed value of standard deviation is also significantly less than the improved value obtained by automatic segmentation. Thus the opportunities to improve the automatic segmentation have not yet been exhausted.

Based on our first results on optimized approximations obtaining for standard and other images followed by their improvement, we venture to assert that for an image approximations which are equalized in segment number, the increase in the value of the standard deviation as a rule is accompanied with the smearing of some visually observed objects (Fig. 4).

16.47 12.67

Figure 4: Optimized image approximations with 99 segments.

Fig. 4 by example of standard Lenna image demonstrates the approximations consisting of equal number of segments. The values of *StdDev* are written under the approximations.

The approximation on the right in Fig. 4 characterized by a smaller value of standard deviation looks more similar to the original than the approximation on the left, characterized by a larger value of standard deviation. The latter is also characterized by better visual quality. With regard to the visual quality of an image approximation it should be noted, that the loss of those or other objects may improve the visual quality of remaining ones though implies some increase in standard deviation value.

5. CONCLUSION

It should be noted that Mumford–Shah model essentially involves the generation of a multilevel hierarchical partitions of an image into the segments of computed shapes, which at first sight, requires the multiple iterations and laborious optimization of algorithms especially in speed. However, the problem of direct program speedup owing to reducing of the iteration number is solved quite simply in the algorithms of synchronous segments merging over the whole image as in version SLBM (Synchronous locally best merging) of the model [4]. But, as rightly noted by the authors of [4], in this case the optimality of approximations may be disturbed due to deviation from the adopted in a model technique of generation of optimized approximations by successive segment merging. This obstacle is avoidable by applying a special data structure that supports the storage of a hierarchy of approximations in lesser part of RAM and provides the conversion of one hierarchy into another to obtain the desired hierarchical sequence of partitions. The applied scheme of optimized computing and our variant of mentioned version of the model providing the generation of mostly binary segment hierarchy and the increasing of processing speed up to over half a million pixels per second is planned to outline in the report.

Probably just the laboriousness of programming restrains the widest propagation of the Mumford–Shah model. Nevertheless, at the present level of computer processing of digital image the segmentation problem, consisting in accurate definition of segmented image, is more practical than theoretical. The theoretical solution is provided, at least, in the Mumford–Shah model that treats the segmented image as an optimal piecewise constant image approximation.

The main limitation of the practical implementation of the Mumford-Shah model consists in that traditional methods for generating of optimized approximations are often restricted to utilization of iterative segment merging. Since the iterative segment merging is insufficient to obtain strictly optimal approximations, the practical results of Mumford–Shah segmentation do not quite match the desired optimal results. In this case, the optimized approximations essentially depend on the used set of segmentation algorithms and are characterized by excessive values of functional. Functional values themselves are quite inactively used in practical calculations. Meanwhile, they seem to be necessary to compensate the effect of variability of image content and object representation, as well as a number of other factors affecting an image partitioning into a relatively small number of segments by means of iterative algorithms.

Probably the standard deviation should be perfectly investigated as the basic functional before the others functional assumptions. Utilization of active contour methods in frameworks of the Mumford–Shah model improves the segmentation results which assessed by the values of the standard deviation, but causes the renewal of computational challenges that arise at a new level. Overcoming these challenges to obtain the optimal image approximations that characterized by reliably minimal values of the standard deviation and more complete disclosure of capabilities of the Mumford-Shah model is the subject of further research. Particularly we focus on lower estimation of attainable minimum of standard deviation for each number of segments, the development of deterministic algorithms for fast generating of optimal approximation series and the generalization of data structures for storage and transformations of a sequence of overlapping image partitions.

In the report additionally to the example of the standard image Lenna the experimental results are illustrated in more detail by the example from the Berkeley Segmentation Dataset and Benchmarks (http://www.eecs.berkeley.edu/Research/Projects/CS/vision/bsds/).

6. ACKNOWLEDGMENTS

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