

# Optimization of Numerically Controlled Five Axis Machining Using Curvilinear Space Filling Curves

Stanislav Makhanov

School of Information and Computer Technology

Sirindhorn International Institute of Technology Thammasat University, Bangkok, Thailand  
[makhanov@siit.tu.ac.th](mailto:makhanov@siit.tu.ac.th)

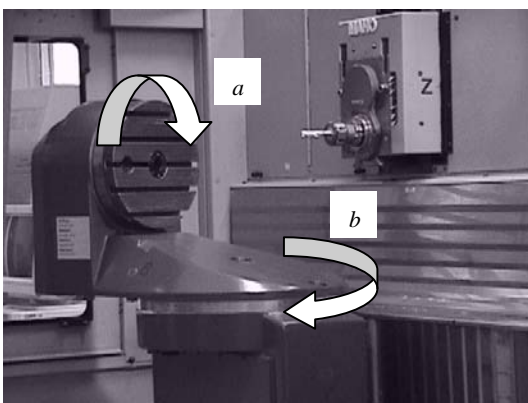
## Abstract

The paper presents optimization and visualization methods for tool path planning of 5 axis milling machines. The first method is based on the grid generation techniques whereas the second method exploits the space filling curve technologies. Combination of the two techniques is superior with regard to the conventional methods and with regard to the case when the two methods are applied independently

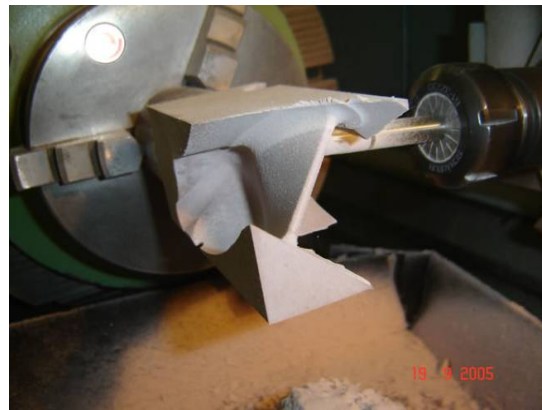
**Keywords:** CAD/CAM, CNC machines, grid generation, space filling curves .

## 1. INTRODUCTION

Milling machines are programmable mechanisms for cutting industrial parts. The axes of the machine define the number of the degrees of freedom of the cutting device. Five axes provide that the cutting device (the tool) is capable of approaching the machined surface at a given point with a required orientation. The machines consist of several moving parts designed to establish the required coordinates and orientations of the tool during the cutting process (see Figure 1 and Figure 2). The movements of the machine parts are guided by a controller which is fed with a so-called NC program comprising commands carrying three spatial coordinates of the tool-tip and a pair of rotation angles needed to rotate the machine parts to establish the orientation of the tool. One may think about such a machine as a 3D plotter with five degrees of freedom.

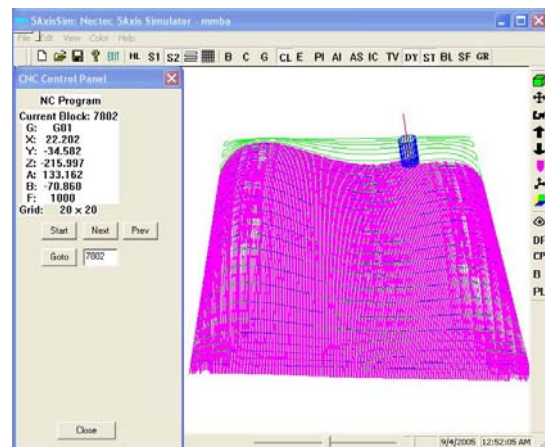


**Figure 1:** Five-Axis milling machines MAHO600E (Deckel Maho Gildemeister) *a* and *b* are the rotation axis



**Figure 2:** MAHO600E during cutting operations

The tool path is a sequence of positions arranged into a



**Figure 3:** Zigzag tool path and the machining strips in the workpiece coordinate system

structured spatial pattern. The conventional engineering patterns are the zigzag and the spiral (Figure 3).

Tool path planning for five axis machining requires a multi criteria optimization governed by estimates of the difference between the required and the actual surface. Additionally, the criteria vector may include the length of the path, the negative of the machining strip (strip maximization), the machining time, etc

[8], [10] . Besides, the optimization could be subjected to constraints [11]

## 2. CURVILINEAR SPACE FILLING CURVES APPROACH

The most popular space filling curve (SFC) for tool path planning is the recursive Hilbert's curve [7] considered for numerous applications including the tool path planning [6].

Cox et al. [5] used various forms of SFCs such as the Moore's curve. However, Hilbert's curve is particularly appealing in tool path planning as its local refinement property can be used to adaptively increase the density of the path only where necessary. However, each local refinement of the tool path based on the Hilbert's curve increases the tool path density in the refined region by a factor of 2 resulting in a lower machining efficiency due to the increased total path length. Besides, the Hilbert's curve has an undesirable property that it leads to a path, where the tool is frequently changing directions which slows down the machining process and produces large kinematics errors. Adaptive SFC [1] has less turns, however, a basic rectangular grid used to construct the adaptive SFCs is often inefficient since a small step between the tracks could be required only in certain areas. The grid is also inefficient in treating complex geometries appearing in the case of the so-called trimmed surfaces having the boundaries created by intersections with other surfaces.

On the other hand the above geometrical complexities and sharp variations of the surface curvature have been proven to be successfully treated by numerically generated curvilinear zigzag tool paths obtained from adaptive grids topologically equivalent to the rectangular grids. In [9] a modification of a classic grid generation method based on the Euler-Lagrange equations for Winslow functional [4] has been adapted to the curvilinear zigzag tool path generation. The zigzag tool path is constructed by solving numerically Euler-Lagrange equations for a functional representing desired properties of the grid such as smoothness, adaptivity to the boundaries and to a certain weight (control) function [2]. A similar idea to use a Laplacian curvilinear grid was suggested independently in [3]. However, these techniques have several major drawbacks. Chief among them is slow convergence for complicated constraints. Besides, the approach requires an equal number of the cutter contact points on each track of the tool. Therefore, if the kinematics error changes sharply from track to track, the method may require an excessive number of points.

This paper introduces a new modification of the grid refinement which fits better in the framework of tool path optimization and is designed specifically for the SFC generation. The method does not require equal number of points on each track. It automatically evaluates the number of the required grid lines. As opposed to the preceding approach, where the weight function represents either the kinematics error or an estimate of the kinematics error (such as the surface curvature or the rotation angles), the proposed algorithm iteratively constructs an adaptive control function designed to represent the scallop height constraints. Additionally, instead of the Winslow functional the new optimization is based on the harmonic functional derived from the theory of harmonic maps. The functional not only provides the smoothness and the adaptivity but under certain conditions guarantees the numerical convergence. Finally, this approach merges with the SFC techniques. In this case, the grid is not converted to the tool path

directly. Instead, it becomes the *basic grid* required for the SFC generation. With this modification, the curvilinear space filling curve (CSFC) tool path can be constructed for surfaces with complex irregular boundaries, cuts off, pockets, islands, etc. Besides, the adaptive grid allows to efficiently treat complex spatial variability of the constraints in such a way that the SFC is created on a grid having the small cells only where necessary.

Finally, the combination of the two techniques is superior with regard to the case when the two methods are applied independently. A variety of examples are presented when the conventional methods are inefficient whereas the proposed algorithms allow constructing the required tool path with the length close to the minimal. The numerical experiments are complemented by the real machining as well as by the test simulations on the Unigraphics 18.

## 3. TOOL PATH GENERATION FOR A SURFACE WITH CURVILINEAR BOUNDARIES AND POCKETS

This example demonstrates the use of the CSFC to construct tool paths to machine surfaces with complex irregular boundaries, cuts off, and islands. Consider a surface shown in Figure 4(a). Figure 4(b) shows the corresponding CSFC and Figure 4(d) the machining result obtained with the use of the solid modeling engine of the Unigraphics. The surface has been machined by a flat-end tool of radius 3 mm and the machined surface tolerance of 0.05 mm. Consequently, the method is capable of creating tool path for surfaces with complex non rectangular boundaries and islands.

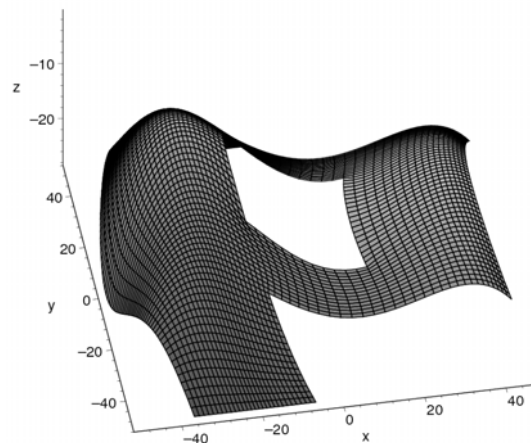


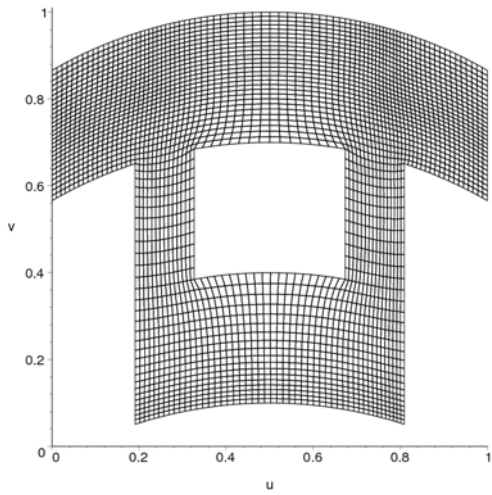
Figure 4 (a): The machined surface

## 4. POINT MILLING OF INDUSTRIAL IMPELLER

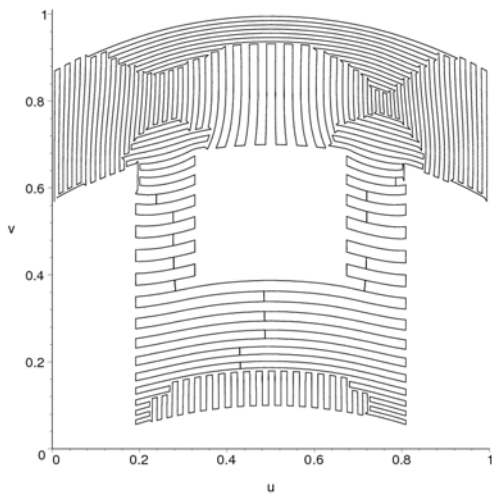
Frequently, the blades of the impellers (Figure 5) are produced by the so-called five-axis *swarf milling* made by a side of the tool. In this case the contact between the workpiece and the cutter is characterized by a contact line rather than a contact point. However, the cutter contact line may lead to large errors. This example demonstrates machining of the blade by 5-axis by (more accurate) *point milling* with the use of CSFC .

In order to demonstrate the advantages of the proposed method, the geometrical complexity of the example blade is increased as

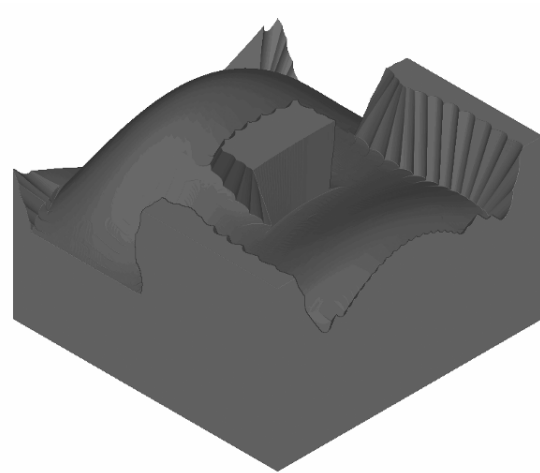
follows. We assume that the blade is broken along a prescribed curve and requires restoration so that we will cut only the missing part of the blade.



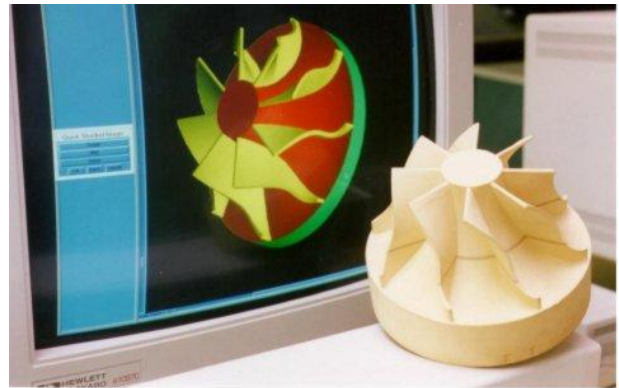
**Figure 4 (b):** The curvilinear grid



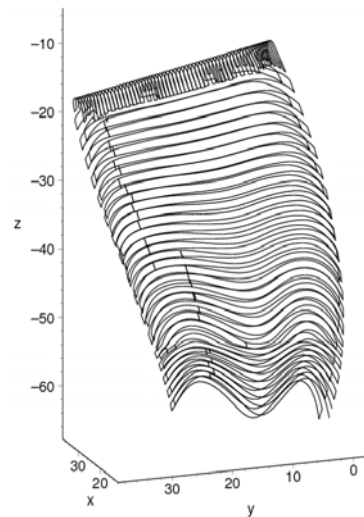
**Figure 4 (c):** The curvilinear space filling curve



**Figure 4 (d):** Virtual machining in Unigraphics



**Figure 5:** An industrial impeller



**Figure 6(a):** CSFC for the "broken blade"



**Figure 6(b):** Virtual machining of the “broken blade”



**Figure 6(c):** Prototype blade (wood)



**Figure 7:** Blade for turbo machinery

The corresponding CSFC tool path is shown in Figure 6(a). A virtual cutting using the proposed CSFC tool path is shown in Figure 6(b) whereas a real prototype of the blade (wood) in Figure 6(c) For demonstration purposes the size of scallops has been

chosen so that the CSFC is visible on the blade surface. Finally, Figure 7 shows an industrial blade suitable for the turbo machinery produced with the proposed method.

## 5. CONCLUSION

Numerically generated adaptive curvilinear grid is introduced to replace the rectangular grid for construction of the space filling tool path for 5 axis machining. With this modification the CSFC can be constructed for surfaces with complex irregular boundaries, cuts off, pockets, islands, etc. The CSFCs are applicable to produce industrial blades for turbo machinery.

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