

# Two Approaches for Noise Filtering in 3D Medical CT-Images

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## Abstract

Noise in 3D computer tomography (CT) images is close to white and becomes large when patient radiation doses are reduced. We propose two methods for noise reduction in CT images: 3D extension of fast rank algorithms (Rank-2.5D) and 3D extension of a non-local means algorithm (NLM-2.5D). We call both our algorithms “2.5D” because the 3-dimensional NLM algorithm is slightly asymmetric by slice axes, while our Rank algorithms, being fully symmetric mathematically and by results, have some implementation asymmetry. A comparison of the methods is presented. It is shown that NLM-2.5D method has the best quality, but is very computationally expensive: its complexity quickly rises as a function of the neighborhood size, while Rank-2.5D only shows a linear growth. Artificial test sequences are used for signal-to-noise performance measurements, while real CT scans are used for visual assessment of results.

**Keywords:** *medical imaging, CT, DICOM, filtering, enhancement, noise reduction, denoising, 3D image processing.*

## 1. INTRODUCTION

The problem of noise reduction in digital images has a long history. The first algorithms were linear filters [7], such as convolutions with a low-pass window function (rectangular or Gaussian), frequency-domain filtering, Wiener filtering. The problem with linear methods is inevitable loss of quality of the image: loss of sharpness, blurring of edges, ringing effects.

The next wide class of image filtering methods has been introduced in [15], they are called rank algorithms. The most well-known of these methods is the median filter, which has fast computational algorithms [3], [14], [6]. Median filter preserves sharp edges, but rounds the corners in the image. Recently we have proposed fast algorithms for other types of range filters [10], [11]. Before their existence, practical applications of such filters were limited. Unlike median filtering and linear methods, rank algorithms reduce the contrast of edges without blurring or changing their shape. A well-known bilateral filtering algorithm can be roughly considered as being a rank filter too. If the existing Gaussian spatial and range kernels in [12] are replaced with rectangular kernels, a so-called  $\epsilon_r$  averaging algorithm is produced [15]. The only fast algorithms known for the bilateral filter are approximating algorithms.

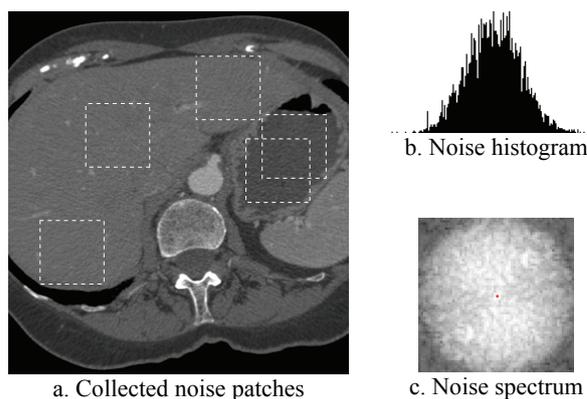
Most recent works in noise reduction show the advantage of methods which average pixels depending on their neighborhood statistics, not just pixel values [1], [2]. The first method, known as non-local means, calculates averaging weights using similarity of pixel neighborhoods (patches). The second one is more complex and consists of two stages. On the first stage a rough frequency-domain filtering is performed to facilitate the search of similar patches. The second stage is the joint filtering of groups of similar patches from the source image. In [2] there

are comparisons of the proposed method with 5 other algorithms.

This paper focuses on noise reduction in computer tomography (CT) scans. CT images capture the density of a sequence of slices of a human body. These slices are obtained with a small fixed stride, which is perpendicular to the slice plane and is comparable with the pixel size in each slice. Together they represent a 3D set of data, so traditional 2D methods of image denoising are less than optimal because they fail to exploit a high degree of data correlation between slices. Independent noise reduction in each slice may cause difference in color and position of edges of objects between slices. Our main objective is to reduce noise and avoid the loss of small low-contrast image areas. Such areas may contain symptoms of an illness and it is important to maintain sharp edges and prevent the loss of information for them. This requirement limits the use of median filtering and other algorithms based on the image blurring as they may shift the edges.

Noise is always present in CT images. Interestingly, as the noise reduction methods advance over time, one can expect even noisier raw data to be coming from the scanner, because it means lower radiation dose for the patient. Specialized algorithms for denoising of CT images are an area of active research [9], [13].

The noise spectrum in our images is close to white: slightly low-pass, but without evident directionality (Figure 1c). The amplitude p.d.f. is close to Gaussian (Figure 1b). This justifies application of standard noise reduction methods, most of which have been formulated for additive white Gaussian noise. Another study of noise in CT images is given in [4], [5].



**Figure 1:** Analysis of noise in our CT images.

This paper proposes two approaches to noise reduction. A higher-quality algorithm is based on a non-local means [1] adapted to a 3-dimensional image data. We are using a fast, but not quite symmetric variant (we call it NLM-2.5D) developed in our previous work for video processing [8]. The second approach (Rank-2.5D) is using the adaptation of fast rank filters which do not blur or shift the edges of the image [10], [11]. The proposed algorithm is fully symmetric in 3 dimensions and

comprises the direct extension of  $\varepsilon_V$  averaging [15] on a 3D space, with an adaptive choice of  $\varepsilon$  in every point of the image. This  $\varepsilon$  is calculated using another rank algorithm. Every slice is processed using information from a *collection* of adjacent slices. The implementation of the algorithm is not symmetric: it is based on sequential processing of 2D slices and 1D filtering across slices. This allows considerable savings in memory, but slows down the computation. The processing time becomes proportional to the number of slices in the collection (neighborhood). A comparison of speed and PSNR quality is carried out on phantom images with an artificially generated noise. Visual results on real medical images are also provided.

## 2. EV - FILTERING

We start with some necessary definitions, following [15]. Consider the current  $v_0$  and some neighborhood  $S$  of pixel  $v_0$  that contains  $N$  pixels. Frequently  $S$ -neighborhood has a square, round or octagonal shape [15], [6].

**Definition 1.** A rank series  $\{v(r)\}$  is a one-dimensional sequence of  $N$  pixels from  $S$  whose elements are sorted in an ascending order with respect to their values:  $\{v(r) \leq v(r+1), r=0..N-1\}$ .

**Definition 2.** Pixel's  $v_R$  rank  $R$  is the position of the  $v_R$  element in the rank series.  $R = rank(v_R)$ .

Let's consider some selected pixel  $v_c \in S$ ; for example we can consider  $v_c = v_0$ ,  $v_c = med\{v(r)\}$ , or  $v_c = mean\{v(r)\}$  as a pivot. It should be noted that under such definition the pivot pixel do not always satisfy conditions  $v_c \in S$  or  $v_c \in \{v(r)\}$ . Then

**Definition 3.**  $\varepsilon_V$  (or EV) neighborhood is a subset of pixels set  $\{v(r)\}$  whose values deviate from the value of the pivot pixel at most by a predetermined quantity  $\varepsilon$ :

$$\varepsilon_V(v_c) = \{v(r) : |v(r) - v_c| \leq \varepsilon\} \quad (1)$$

As it can be seen from the definition,  $\varepsilon_V$  neighborhood average can be treated as a simplified bilateral filter [12], where bilateral filter parts that depend both on distance and on pixel brightness, are represented by rectangular functions instead of Gaussians. For this reason  $\varepsilon_V$  filtering should keep the edges of the objects sharp, assuming the parameter  $\varepsilon$  is properly chosen.

## 3. 3D RANK FILTERING ALGORITHM

It is proposed to use a 3-dimensional  $\varepsilon_V$  filtering algorithm with an adaptive search of parameter  $\varepsilon$  for denoising of CT images. Fast rank algorithms [10] are based on multiscale histogram approach. Either fast rank algorithms or a lazy calculations technique remain the same in a 3D case, though some difficulties occur with column histogram maintaining [6], [11]. Thus if a 2-dimensional image with the size of 512x512 pixels requires maintaining of 512+1 multiscale histograms, then in a 3 dimensional case we will have to maintain 512x512+1=262145 multiscale histograms for a sequence of images with the same size. Taking into account the large size of multiscale histograms with the specific additional information [10], we may see that the number above is at the limit of memory size of 32-bit computers. We propose to use a separable approach for reduction of complexity by processing 2D images. At first,  $\varepsilon_V$  neighborhood average will be calculated

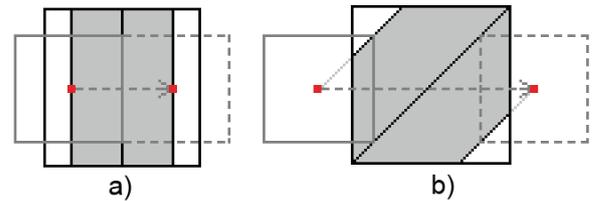
separately with the same pivot for all  $N$  slices that are used for denoising of the current image. Then the 1D variant of  $\varepsilon_V$  filtering will be applied to the result. Really, if the inequality (1) is true for a number of subsets it is true for the union of these subsets too.  $\varepsilon_V$ -averaging means calculation of the ratio of sum of values to number of elements, so it is sufficient to sum up the sums for each subsets and the number elements in each subsets and then calculate their ration once.

For the correct choice of the  $\varepsilon$  parameter we propose to calculate the intensity variance in a square neighborhood of small radius  $R_{disp}$  for each pixel of the image. This step allows determining the uniformity of an area of a specified radius around the pixel. For a flat area with insignificantly varying color the intensity variance value will provide the noise variance.

The choice of  $R_{disp}$  is based on the fact that the noise in CT images is almost non-correlated (white). So the neighborhood can have an arbitrary radius. Since our main goal is to preserve small low-contrast objects, it is essential to choose  $R_{disp}$  in accordance with the size of the object that needs to be preserved. Since the variance will be high on the boundaries of areas with different colors, it is proposed to apply a some kind of minimum filter to the variance image, using the neighborhood of a larger radius. This allows propagating correct values of noise variation to regions with excessive variance.

So the next step is the min-filtering of variance by averaging  $\kappa$  pixels with the smallest rank of the neighborhood of a larger radius. To completely suppress high-variance values on boundaries it is proposed to use the neighborhood of a higher radius  $1.5 \cdot R_{disp}$ . Let us consider possible cases for location of a boundary within the neighborhood. Since the radius of the neighborhood is small enough, we assume that the boundary between the objects inside the sliding window is a straight line.

In the easiest case (Figure 2a) the amount of pixels that represent variance outside the edge area is about 30% of the bigger neighborhood. In the worst case (Figure 2b) the amount of pixels for the variance of the outside-edge area is about 10%. These values can be easily proved by simple geometrical calculations.



**Figure 2:** The area with high variance value for a) the vertical edge; b) the diagonal edge

Variance minimized in such a way corresponds to adaptive  $\varepsilon$  value. In order to control the degree of noise reduction it is proposed to use a multiplier parameter  $M$ . In general the coefficient  $M$  is user-defined, but it is essential to use the range of [2...6] in order to simultaneously suppress the majority of noise pixels and to avoid blurring of the edges.

Then  $\varepsilon_V$  algorithm with the  $\varepsilon$  value that was obtained on the previous step and the  $R_{disp}$  neighborhood radius is performed for each slice. It is important to emphasize that we take not the central intensity value of the current slice but the intensity value of a source image with corresponding coordinates as a

central pixel for  $\varepsilon_V$  neighborhood average algorithm. This condition is important because we apply the algorithm not to a 2D neighborhood, but to the volume region.

To extend the algorithm to a 3D space, it is proposed to calculate the total sum and the total number of elements involved in the averaging for the current pixel of the current slice. As a result, for each pixel of each slice we receive the structure that contains the total sum and the amount of the  $\varepsilon_V$  neighborhood elements.

The last step is  $\varepsilon_V$  filtering for slices. It is proposed to use the minimized variance for the source image multiplied by  $M$  as the value of  $\varepsilon$  for each pixel of the source image. The averaging is performed with only one structure from each slice (i.e. if we use  $N$  slices, only  $N$  elements will take part in averaging). For the pixel  $I(x, y)$  of the source image the necessary structures will be located at the same coordinates in the corresponding slice. The number of slices for processing is defined automatically in accordance with the distance between slices.

Let  $N$  be the number of images (slices) for denoising of one image,  $I_{src}$  be the source image,  $I_i$  be the  $i$ -th image from the array of image slices,  $R_{disp}$  be the radius of the variance calculation window,  $D(I_i)$  be the variance of the  $i$ -th image,  $M$  be the variance multiplier,  $R_{2D}$  be the radius for a 2D spatial  $\varepsilon_V$  filtering. The following algorithm is proposed for CT image denoising.

**Algorithm 1.** 2.5D rank denoising algorithm.

1. **for**  $i := 0, i < N$  **do**
2.     **for** each pixel  $I_i(x, y)$  of  $I_i$  **do**
3.         Compute the variance of  $I_i(x, y)$  with the window radius  $R_{dips}$ ;
4.         Minimize variance  $D(I_i(x, y))$  for  $I_i$  by averaging  $K$  elements with the smallest rank using the window radius of  $1.5 \cdot R_{dips}$ ;
5.         Perform  $\varepsilon_V$  filtering around the current pixel's  $I_i(x, y)$  neighborhood to obtain the total sum and total count of elements for  $I_i(x, y)$ .  $\varepsilon(x, y) = M \cdot D(I_i(x, y))$ , the window radius is  $R_{dips}$  and  $I_{src}(x, y)$  is taken as the central pixel of the neighborhood;
6.     **end for**;
7. **End for**;
8. **for** each pixel  $I_{src}(x, y)$  of  $I_{src}$  **do**
9.     Perform 1D  $\varepsilon_V$  filtering with  $\varepsilon(x, y) = M \cdot D(I_{src}(x, y))$  for  $N$  corresponding elements  $I_i(x, y)$ ,  $i = \overline{1, N}$ ;
10. **End for**.

**4. 3D NON-LOCAL MEANS FILTERING**

Bilateral filtering algorithm is well known in image processing for its simplicity and edge-preserving properties [12]. The output pixel value  $I_{out}(x, y)$  is formed as a weighted sum of pixel values from the neighborhood  $\Omega$ :

$$I_{out}(x, y) = \frac{1}{\sum_{i, j \in \Omega} W} \sum_{i, j \in \Omega} W(x, y, j, i) \cdot I(x + j, y + i) \quad (2)$$

The weights  $W$  depend on geometric distance and color difference between pixels  $(x, y)$  and  $(x + j, y + i)$  in order to facilitate averaging of pixels with similar values:

$$W(x, y, j, i) = \exp \frac{j^2 + i^2}{-2\sigma^2} \cdot \exp \frac{(I(x + j, y + i) - I(x, y))^2}{-2\rho^2} \quad (3)$$

Non-local means is a relatively novel method of image filtering that builds upon a bilateral algorithm. A formula (3) for pixel similarity in bilateral filtering considers colors and spatial coordinates of two pixels. In the non-local means algorithm, this formula instead considers *the context* of two pixels [1]. Specifically, instead of comparing values of two pixels, the algorithm compares the content of *image patches*  $v$  around two pixels:

$$W(x, y, j, i) = \exp \frac{\|v(x + j, y + i) - v(x, y)\|_2^2}{-2\rho^2} \quad (4)$$

The squared norm of pixel-wise patch differences in formula (4) ensures that only pixels with a similar surrounding content are averaged together.

The extension of this method on a 3-dimensional space is straightforward. For the standard 2D image processing, the neighborhood  $\Omega$  from formula (2) is a circle or a square around the central pixel. For the 3D filtering, we extend the neighborhood to be a sphere or cube in the 3D image space: it includes several image slices that are adjacent to the processed pixel. A 2-dimensional summation in formula (2) turns into a 3-dimensional summation and calculation of weights in formula (4) is adjusted accordingly: similar patches are searched among the array of several adjacent slices. Since the compared patches are still 2-dimensional, we call this method NLM-2.5D.

For improved speed of calculations we employ an optimization from [8] for sparse update of weights  $W$ .

**5. SYNTHETIC PHANTOM GENERATION**

For testing the noise reduction capability of our algorithms we have generated a synthetic phantom image. We don not use real phantom images because they always contain noise due to CT-imaging generation process. An attempt to shoot a phantom without noise results in high-dose shooting which is radically different from real medical imaging conditions. Our synthetic phantom construction (see Figure 3) is described below. The phantom body is cylindrical. It has two cylindrical "organs" with different radiopacity. These two "organs" are connected by two thin truncated cones ("processes" or "ligaments"), with radiopacity smoothly changing from one organ to another. Each "organ" has one truncated conic "vessel" with different radiopacity. These truncated cone shapes are used for thin objects in order to detect the extent of damage to the phantom along the slice axis in the process of filtering (if any).

We model all "body", "organs", "ligaments", and "vessels" with color  $c_j$  and density function

$$\rho(\vec{\mathbf{r}}) = \exp \left\{ - \left( \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_0}{R} \right)^{2n} \right\}, \vec{\mathbf{r}} = (x, y) \quad (5)$$

which is used smooth object boundaries, mainly to avoid pixelated (aliased) shapes. The center of small objects ("ligaments")

and “vessels”) depends on a slice position  $z : \mathbf{r}_0(z), R(z)$ , so density function depends on all 3 coordinates  $\rho(\mathbf{r}) = \rho(x, y, z)$ . The resulting slice image set is obtained recursively as

$$\begin{aligned} I_0(x, y, z) &= \rho_{body}(\mathbf{r})c_{body} \\ I_j(x, y, z) &= \rho_j(\mathbf{r})c_j + (1 - \rho_j(\mathbf{r}))I_{j-1}(x, y, z) \end{aligned} \quad (6)$$

where large objects are added first. Typical simulated phantom images are presented in Figure 3.

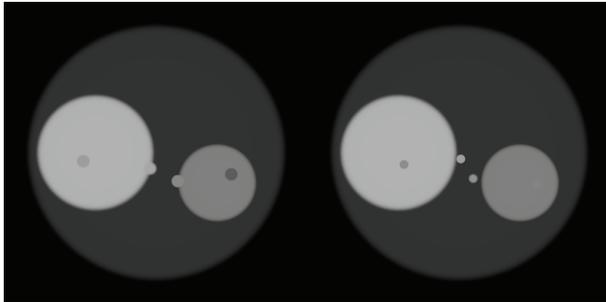


Figure 3: Simulated noise-free phantom images

The sharpness of edges is  $n = 30$  for body,  $n = 20$  for “organs”, and  $n = 4 \dots 12$  for different “ligaments” and “vessels”. The size of each slice is  $512 \times 512$  pixels and the total number of slices is 100. Thin object radii  $R(z)$  are from 5 to 12 pixels.

## 6. RESULTS

Figure 4 shows the results of a phantom CT image denoising. The Gaussian noise which corresponds to the real noise distribution on CT images was added to the synthesized phantom slices. The measured value is the improvement of PSNR in decibels between processed images and the ground-truth noise-free image. Each algorithm has been run with optimal parameters which maximize PSNR for each size of the neighborhood. The optimized parameter for Rank-2.5D was  $M$ , while the optimized parameter for NLM-2.5D was  $\rho$ . The patch size in NLM has been set to  $8 \times 8$  pixels, while the “pixel” size has been set to  $2 \times 2$  pixels.

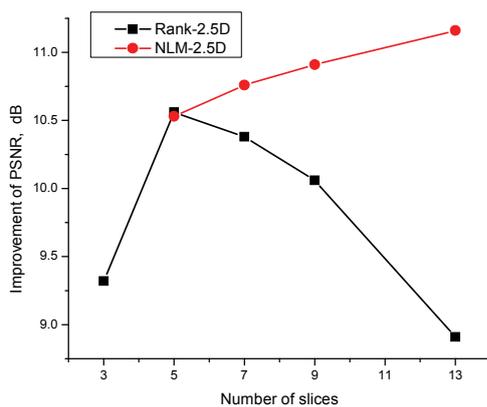


Figure 4: Improvement of PSNR for the phantom image filtered by different algorithms, depending on the neighborhood size  $N$ .

The maximum in the curve for Rank-2.5D algorithm occurs when the neighborhood size is of order of smallest significant object size (“ligaments” and “vessels”). When the neighborhood becomes larger, small objects are suppressed as noise.

It can be seen that the algorithms are able to exploit high degree of correlation between image slices, which is reflected in PSNR and visual quality.

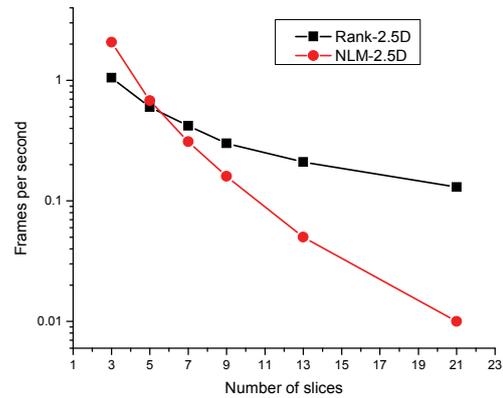


Figure 5: Speed of our C++ implementation of the proposed algorithms on a 2.8-GHz desktop.



Figure 6: Original noisy CT image, result of Rank-2.5D algorithm, result of NLM-2.5D algorithm.

Figure 5 compares the speed of the proposed algorithms on a CT scan with  $512 \times 512$ -pixel slices. It shows that the Rank-2.5D

algorithm has a linear complexity growth depending on the neighborhood size, which makes it suitable for future high-resolution CT scanners. The NLM-2.5D algorithm's complexity grows much faster.

Figure 6 shows the result of real CT image denoising with Rank-2.5D and NLM-2.5D algorithms operating on a neighborhood of  $7 \times 7 \times 5$  pixels (the last '5' being the number of slices). The value of parameter M in Rank-2.5D method has been set to 3.5 for good visual results. This is lower than the value of  $M = 5$  which optimized PSNR in our experiments.

Subjectively, NLM-2.5D approach provides more accurate and clear image while Rank-2.5D leaves some blurred patches in high-contrast areas.

## 7. CONCLUSION

Two methods for 3-dimensional noise reduction in CT images have been presented. The evaluation shows that they are able to effectively exploit the existing correlation between CT slices for improvement of the resulting image quality. The presented Rank-2.5D shows only moderate growth in computational complexity depending on the size of neighborhood. It is easy to see from Figure 5 that when the neighborhood size in slices is larger than 6, Rank-2.5D becomes faster than NLM-2.5D. The maximum quality of Rank-2.5D is achieved when the neighborhood size is approximately equal to the minimal useful object size in the image.

## 8. ACKNOWLEDGMENTS

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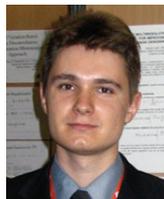
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