Writing a ray tracer

What coordinate system?

- **world space, not eye space**
  - viewing transformation would only simplify 1st ray
  - objects change shape in perspective, so intersections are harder
  - perspective is not angle-preserving, so bounces are harder

Ray tracing algorithm

```
for each pixel (or sample)
    for each primitive
        test for ray-primitive intersection
    select frontmost intersection
    trace new rays (recursively) in reflection / refraction / light directions
    compute color
```

Two ways to define a geometric primitive

Implicit form $F(x,y,z)\begin{cases} < 0 & \textit{inside} \\ = 0 & \textit{on} \\ > 0 & \textit{outside} \end{cases}$

Parametric form $(\,x(u,v),\, y(u,v),\, z(u,v)\,)$

Example: a plane

implicit

$$ax + by + cz + d = 0$$

$$\mathbf{N} \bullet \mathbf{P} = 0$$

parametric

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = u \begin{bmatrix} x_u \\ y_u \\ z_u \end{bmatrix} + v \begin{bmatrix} x_v \\ y_v \\ z_v \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

Ray-surface intersection

$$f(x,y,z) = 0$$

$$x = x_0 + t\Delta x$$
$$y = y_0 + t\Delta y$$
$$z = z_0 + t\Delta z$$

$$f^*(t) = 0$$

1. **Substitute parametric form of ray equation into implicit form of surface equation**

2. **Find *positive*, *real* roots**

Univariate root finding

- **closed form solution**
- **Newton's method**
- ***regula falsa***
- **interval methods**

(from Hanrahan)

Ray-sphere intersection

Sphere $(x-a)^2+(y-b)^2+(z-c)^2-r^2=0$

Ray $x = x_0 + t\Delta x$

$y = y_0 + t\Delta y$

$z = z_0 + t\Delta z$

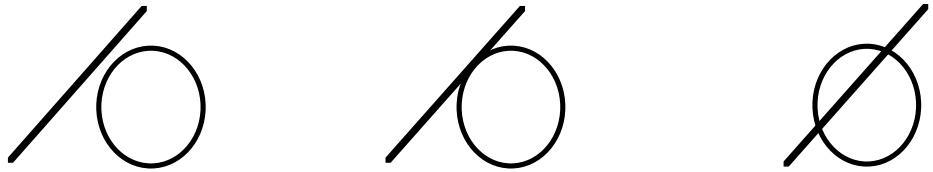
Substituting gives $At^2 + Bt + C = 0$

where $A = \Delta x^2 + \Delta y^2 + \Delta z^2$

$B = 2(\Delta x(x_0 - a) + \Delta y(y_0 - b) + \Delta z(z_0 - c))$

$C = (x_0 - a)^2 + (y_0 - b)^2 + (z_0 - c)^2 - r^2$

Solution $$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$



If discriminant **<0** **=0** **>0**

Point of intersection $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + t \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}$$

Ray-plane intersection

Plane $ax + by + cz + d = 0$

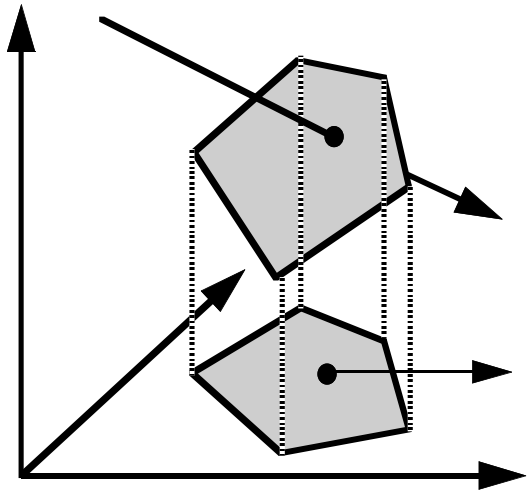
Ray
$$x = x_0 + t\Delta x$$
$$y = y_0 + t\Delta y$$
$$z = z_0 + t\Delta z$$

Solution $$t = -\frac{ax_0 + by_0 + cz_0 + d}{a\Delta x + b\Delta y + c\Delta z}$$

If denominator = 0, ray is parallel to plane

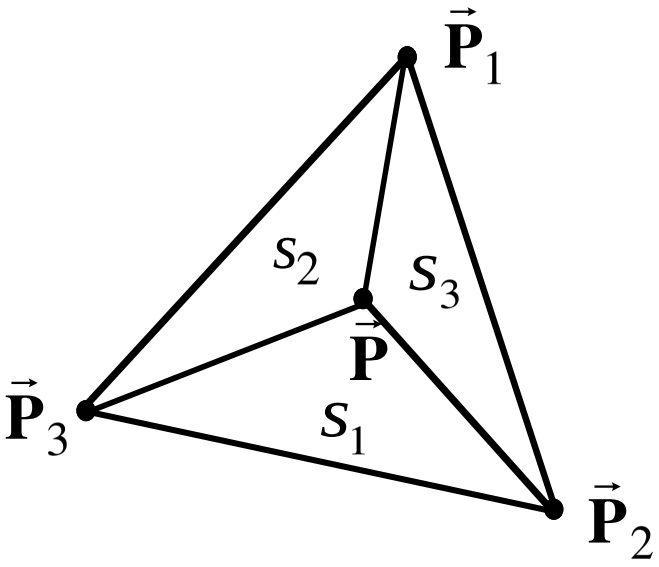
Ray-polygon intersection

1. Find intersection with plane of support
2. Test whether point is inside 3d polygon
    a. Project onto xy plane (actually drop largest of a,b,c in N)
    b. Test whether point is inside 2d polygon



(from Hanrahan)

Ray-triangle intersection



Barycentric coordinates

$$\vec{\mathbf{P}} = s_1\vec{\mathbf{P}}_1 + s_2\vec{\mathbf{P}}_2 + s_3\vec{\mathbf{P}}_3$$

$$s_1 + s_2 + s_3 = 1$$

Solution

$$s_1 = \frac{\begin{vmatrix}\mathbf{P} & \mathbf{P}_2 & \mathbf{P}_3\end{vmatrix}}{\begin{vmatrix}\mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3\end{vmatrix}}$$

$$s_2 = \frac{\begin{vmatrix}\mathbf{P}_1 & \mathbf{P} & \mathbf{P}_3\end{vmatrix}}{\begin{vmatrix}\mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3\end{vmatrix}}$$

$$s_3 = \frac{\begin{vmatrix}\mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}\end{vmatrix}}{\begin{vmatrix}\mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3\end{vmatrix}}$$

Inside criteria

$$0 \le s_1 \le 1$$

$$0 \le s_2 \le 1$$

$$0 \le s_3 \le 1$$

(from Hanrahan)

Other implicit surfaces

algebraic surfaces

- **implicit surface of degree d**
  - linear plane
  - quadrics sphere, cone, cylinder, paraboloid, hyperboloid
  - quartics torus
  - superquadrics, etc.
- **substitute ray equation, then find roots as before**

tensor product surfaces

- **implicitization**
  - degree m in u, n in v $\rightarrow$ degree 2mn implicit surface
  - bilinear patch $\rightarrow$ degree 2 implicit surface
  - bicubic patch $\rightarrow$ degree 18 implicit surface