Radiosity with Textures, Specular Reflection and Transmission

Thomas Leiss
Institute of Computer Graphics
Visualization and Animation Group
Vienna University of Technology

Peter Ferschin
Institute of Computer Aided
Planning and Architecture
Vienna University of Technology
ferschin@osiris.iemar.tuwien.ac.at

Werner Purgathofer
Institute of Computer Graphics
Visualization and Animation Group
Vienna University of Technology
purgathofer@cg.tuwien.ac.at

Abstract

The radiosity algorithm introduced by Goral et al. [Goral84] was designed to simulate the transfer of light in purely diffuse environments. This limitation prevents the possibility to render objects with materials like glass or metal. To overcome this problem, extensions can be added to the basic algorithm. This paper describes extensions to the traditional radiosity algorithm that include texture mapping, specular reflection and specular transmission.

Keywords: Rendering, Radiosity, Textures, Specular Reflection, Transmission

Radiosity

Radiosity is defined as the total energy radiated from a surface per unit time in W/m^2. Only the part of visible light is considered in computer graphics. To calculate the radiosity in a virtual environment, two assumptions must be met:

1. all surfaces are divided into finite elements (patches) with constant radiosity
2. all emissions and reflections are purely diffuse

Each patch is represented by a node on the patch. If the first assumption can't be met on a patch, the patch must be subdivided. The radiosity of a patch *i* is defined as:

$$B_i = E_i + \rho_i \sum_{j=1}^{n} B_j F_{ij}$$

equation 1

where:

B_i	radiosity of patch *i*
E_i	emission of patch *i*
ρ_i	reflectance of patch *i*
n	number of patches in the scene
F_{ij}	form factor

The form factor F_{ij} describes the fraction of radiosity, that patch *i* receives from patch *j*. F_{ij} is defined as:

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{cos(\theta)cos(\theta')}{\pi r(x,x')^2} V(x,x') dA_j dA_i$$

equation 2

The form factor determines the geometrical relationship for each point on surface *i* to each point on surface *j*. Therefore the visibility function $V(x, x')$ is needed. $V(x, x')$ is 1, if the two points see each other, otherwise it is 0. The points can also be seen as differential surfaces, with no area.

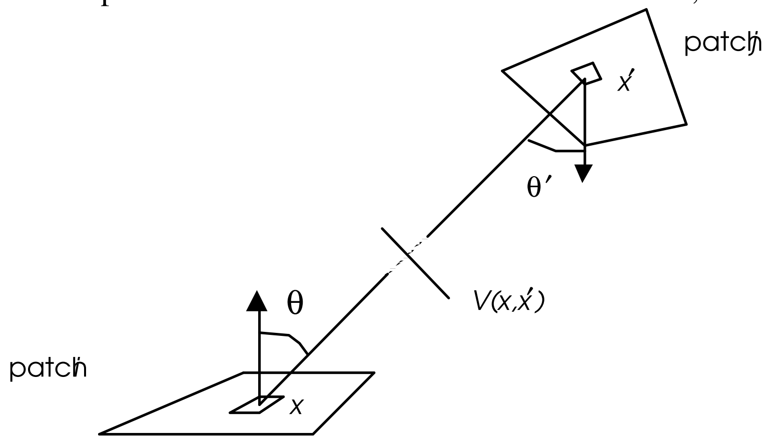

figure 1

The calculation of the double integral is difficult to perform, so simplification is necessary. If it is assumed that the form factors for all *x* on patch *i* are nearly constant, the outer integral can be omitted. This assumption can be met if patches are small and flat.

$$dF_{dA_i \rightarrow A_j} = \int_{A_j} \frac{cos(\theta)cos(\theta')}{\pi r(x,x')^2} V(x,x')dA_j$$

equation 3

There exist several ways to substitute the inner integral, the most important can be found in [Cohen93] or [Sillion94]. In this work two different methods are used:

1. **Analytical form factor computation:**
   The form factor depends on distance, area and orientation of the two patches and can be computed with a contour integral as further described in [Baum89]. The analytical form factors are very accurate but lack of any visibility testing. Therefore visibility must be tested in a separate algorithm. This was achieved by shooting rays and testing for intersection.

2. **Disc form factor computation:**
   Instead of an exact calculation of the projection, the patch can be substituted by a disk with the same area [Wallace87]. The main problem of this method is that this approximation delivers bad results for a small distance or a large area of the patch *j*. So disk form factors should only be used if $\Delta A_j << r^2$ is true.

Solving the radiosity system

The radiosity equation (equation 1) describes a system of *n* equations that can also be written in matrix form. The matrix solution can be computed with common mathematical methods like Gauss-Seidel iteration. In this method, an initial value is guessed, which should be close to the sought result. For the radiosity matrix, the emissions E_n can be used as such values for B_n. The equation system will be calculated based on these values. The new results are the base for the next iteration. This will be repeated until sufficient accuracy is reached. The drawback of this computation is that you have to calculate all lines of the matrix until each patch receives energy. Instead of calculating the radiosity, one patch receives from all others (gathering), it is better to distribute the radiosity of one patch to the others (shooting) [Cohen85]. By choosing the patch with the most unshot radiosity, the system is reaching the final result much faster. This patch is called the shooter.

Instead of representing a patch by one node, it is better to use all vertices of the patch. This allows the use of a Gouraud shading method to calculate the intermediate values.

So far only diffuse surfaces with linear shading could be used in scenes. One major improvement for a realistic impression of a virtual environment is the use of textures.

Textures

Textures can be represented in two or three dimensions. To put a texture onto a surface, a function $f_t(x,y,z)$ is needed, which returns the corresponding color of the texture for a point (x,y,z) of the surface. As color changes often within a texture, it would be necessary to use a huge amount of linear shaded patches to display it properly. Instead of subdividing the patch, an average color is used for the radiosity computation. The actual color will be inserted in the rendering step through the following equation:

$$B(x,y,z) = B_d(x,y,z)\frac{f_t(x,y,z)}{c_{avg}}$$

equation 4

B_d radiosity calculated with local shading
c_{avg} average color of the texture.
$f_t(x,y,z)$ texture function

Specular reflection

Normally only ideal specular reflection is taken into account. There are two ways to include specular reflection into the radiosity solution:

1. The main reason why surfaces had to be purely diffuse, was the fact that the direction, from where reflected light comes from, is not needed to know. Therefore only the amount of energy has to be stored. This is not sufficient for specular reflection. When a patch distributes it's specular received energy, it must know where it came from. Global cubes are an approach to simply store this information [Immel86]. Similar to textures the intensity changes very quick within short range, so a huge amount of data would be needed to achieve enough accuracy.

2. Instead of storing the information, the energy can be traced until it reaches a diffuse only surface. This can be achieved by mirroring the whole scene. The memory needs would be very high to accomplish the creation of the virtual scenes, especially when the virtual scenes have to be mirrored again. Sillion et al. restrict specular reflection to plane patches [Sillion91]. For plane mirrors, only the shooter must be mirrored and the visibility can be calculated in the real scene. This calculation is divided into steps, which compute the visibility between the patch and the mirror, then between the mirrors if the energy will be reflected more than once and finally between the last mirror and the shooter. This must be done for all rays, which test the visibility.

$$B_i = E_i + \rho_i^d \sum_{j=1}^{n} \left[B_j F_{ij} + \rho_i^s \sum_{p=1}^{n} B_p F_{ijp}^m \right]$$

equation 5

ρ_i^d	diffuse reflection coefficient
ρ_i^s	specular reflection coefficient
F_{ijp}^m	form factor from i to p, reflected by j

Transmission

Diffuse Transmission

The calculation of perfect diffuse transmission is similar to perfect diffuse reflection, no directional information has to be considered.

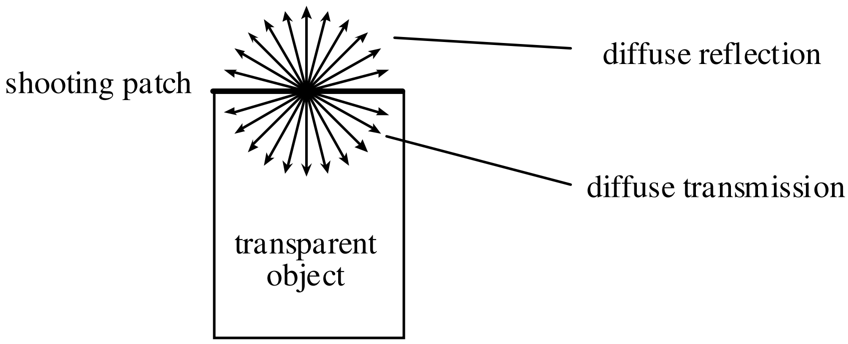

figure 2

The radiosity equation can therefore be expanded to:

$$B_i = E_i + \rho_i \sum_{j=1}^{n} B_j F_{ij} + \tau_i \sum_{j=1}^{n} B_j T_{ij}$$

equation 6

τ_i	diffuse transparency coefficient
T_{ij}	transmission form factor

Specular Transmission

The problem in computing specular transmission is, that refraction causes view dependent bending. Using virtual shooters is more complicated. Certain assumptions have to be fulfilled:

- Distance between shooter and receiving patch is large
- Shooters are small
- Patches are planar
- Bending caused by refraction is small

The first two assumptions are needed for the use of disk form factors. The last three are used to generate a virtual shooter for transmitting surfaces.

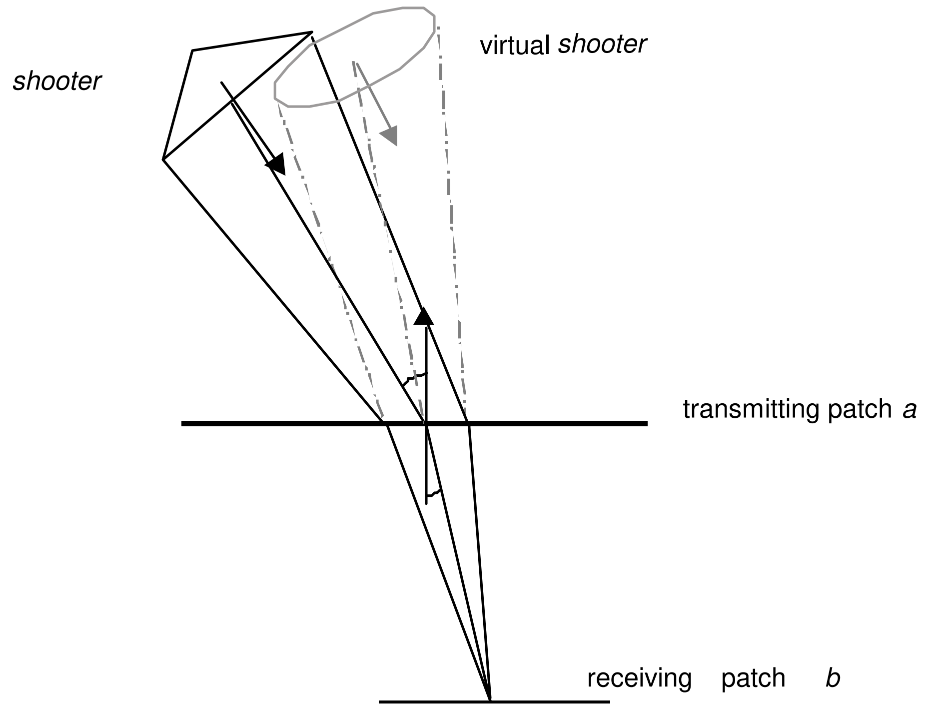

figure 3

To add the influence of specular transmission to the radiosity equation the following extensions have to be made:

$$B_i = E_i + \rho_i \sum_{j=1}^{n} \left[B_j F_{ij} + \tau_j^s \sum_{p=1}^{n} B_p T_{ijp}^f \right] + \tau_i \sum_{j=1}^{n} \left[B_j T_{ij} + \tau_j^s \sum_{p=1}^{n} B_p T_{ijp}^b \right]$$

equation 7

T^f	transparency form factor on the front side
T^b	transparency form factor on the back side

To calculate the exact form factor for this virtual shooter, it would be necessary to rotate the vertices in consideration of the position of the receiving vertex. This task is not simple for multiple refraction. Instead of rotating all vertices, it is possible to rotate the center and create a disk, with the same area of the patch. The normal vector of this disk is the rotated normal vector of the original shooter. Still this does not solve the problem of determining the exact route between the shooter and the receiving vertex. Therefore rays are shot through the transmitting patch. The possibility of hitting the position of a vertex is low, that is why hits are assigned to the nearest vertex. This causes an imaginary subdivision of the patch, which is not represented by the mesh.

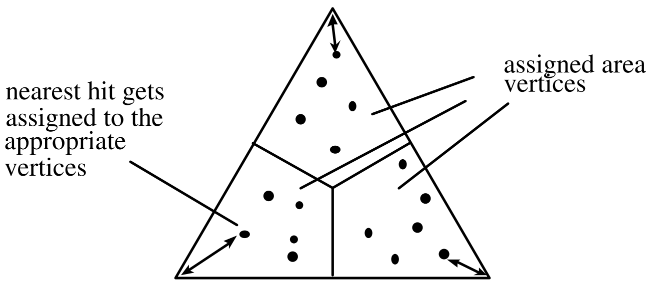

figure 4

A vertex can be used by more than one patch, this increases the assigned area of the vertex.

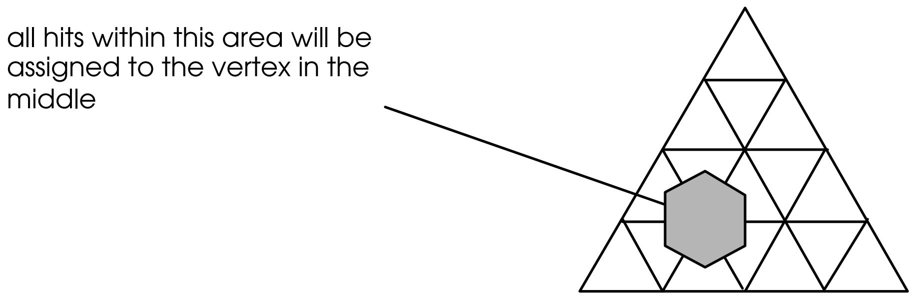

figure 5

If more than one hit is assigned to a vertex, only the nearest will be taken into account. Therefore the energy and distance (position) of the hit is stored, that it can be removed if a nearer hit occurs. In addition the number of hits is stored, which is needed for subdivision. There are three subdivision criterions:

Type 1: Not all vertices got a hit assigned.

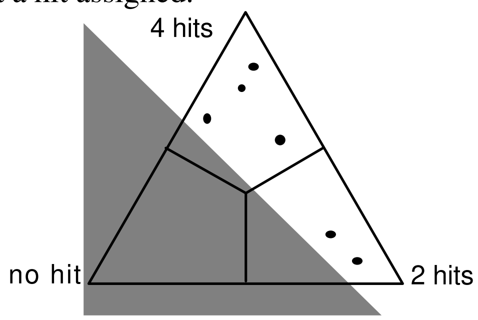

figure 6

The danger of this criterion lies within the hit density. If too few rays are shot, many patches will be divided, although the shooter is completely visible.

Type 2: One vertex received far less hits assigned compared to the other vertices on the patch.

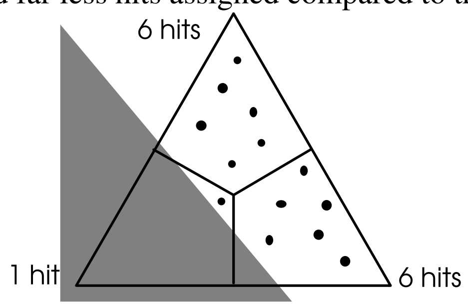

figure 7

If the density of hits is high enough, shadow edges can be detected by comparing the hit counts of the vertices.

Type 3: Nearest hit is too far away.

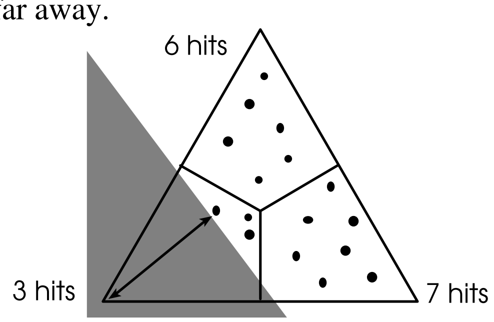

figure 8

If the nearest hit is too far away, the patch has to be subdivided. Again the density of hits must be high. After subdivision, rays are shot again. This will be repeated until no patches are subdivided. When a patch is subdivided during this iteration, assigned hits must be reassigned because maybe one of the new nodes is closer to a hit.

If all mentioned effects (specular reflection, diffuse and specular transmission) have to be calculated, the radiosity equation expands to:

$$B_i = E_i + \rho_i^d \sum_{j=1}^{n} \left[B_j F_{ij} + \tau_j^s \sum_{p=1}^{n} B_p T_{ijp}^f + \rho_j^s \sum_{p=1}^{n} B_p F_{ijp}^f \right] + \tau_i^d \sum_{j=1}^{n} \left[B_j T_{ij} + \tau_j^s \sum_{p=1}^{n} B_p T_{ijp}^b + \rho_j^s \sum_{p=1}^{n} B_p F_{ijp}^b \right]$$

equation 8

Solution drawbacks

- The visibility is tested with rays only, therefore a high shooter subdivision is needed.
- Small patches are hard to hit, it can only be achieved with a number of rays.

Results

The following images will demonstrate all extensions that have been implemented in this work.

The image quality depends on two factors:

- the quality of the initial mesh
- the selected quality parameters (subdivision depth, number of iterations)

Implementation of the radiosity system was done on a PC (Pentium CPU, 200Mhz, 96 MB RAM).

Three Boxes with Mirror Effect

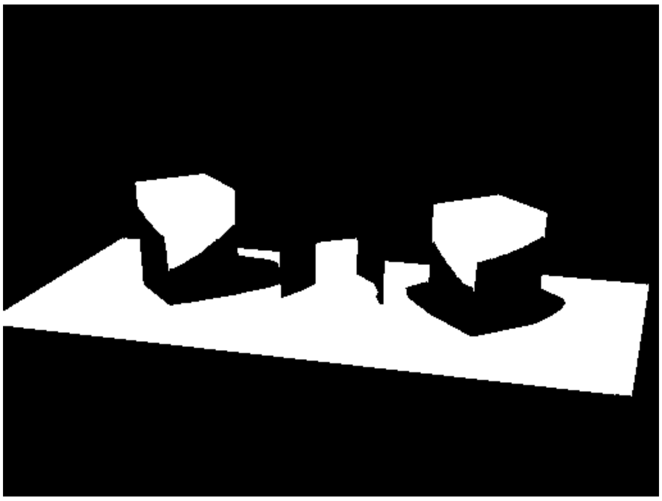

figure 9, calculation time: 1h 20 min

Three Boxes with Textures

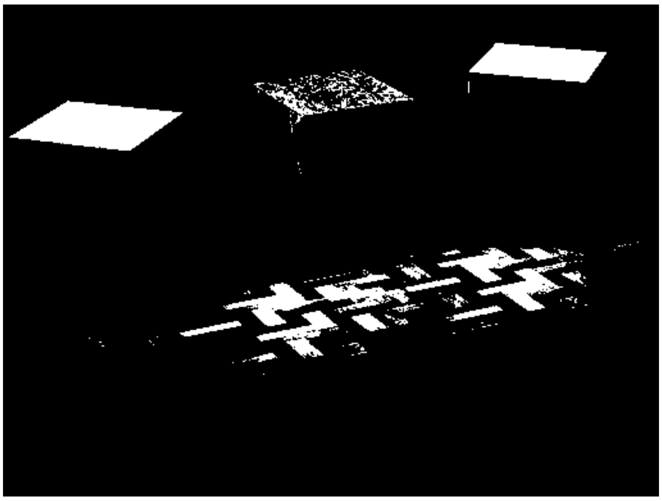

figure 10, calculation time: 1h 5 min

Four Walls with Transparency Effect

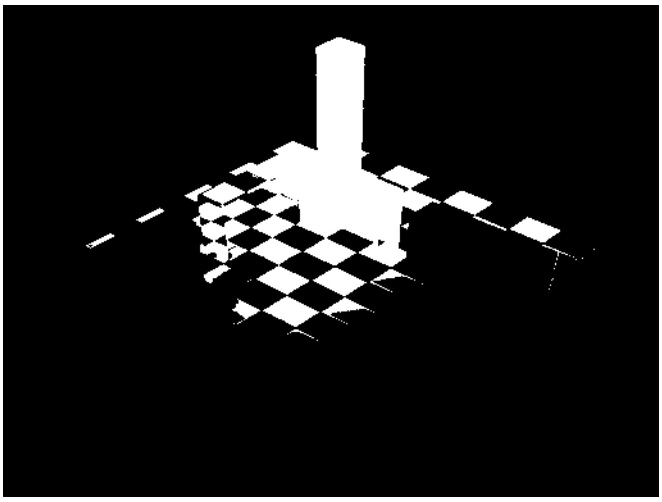

figure 11, calculation time: 16h 23 min

Conclusion

Several extensions to the traditional radiosity algorithm, textures, specular reflection and transmission have been introduced and implemented. These effects enhance the quality of the calculated images significantly although the high amount of rendering time, especially for calculating transparency has to be mentioned. The random selection of rays should be replaced by a more directed approach.

Also traditional acceleration methods, already known from the research of ray tracing should be used in future enhancements.

Another, more general problem results in the high costs of subdividing the environment to gain a good image quality. Additional research is necessary to prevent typical radiosity artifacts as shadow and light leaks.

References

[Baum89] Baum, D. R., Rushmeier, H. E., Winget, J. M.: *Improving Radiosity Solution Through the Use of Analytically Determined Form-Factors*. In Computer Graphics SIGGRAPH ¥89, 23(3), 325-334, ACM SIGGRAPH New York, 1989

[Baum91] Baum, D. R., Mann, S., Smith, K. P., Winget, J. M.: *Making Radiosity Useable: Automatic Preprocessing and Meshing Techniques for the Gerneration of accurate Radiosity Solution*. In Computer Graphics SIGGRAPH ¥91, 25(4), 51-60, ACM SIGGRAPH New York, 1991

[Cohen85] Cohen, M. F., Greenberg, D. P.: *The hemi-cube: A radiosity solution for complex enviroments*. In Computer Graphics SIGGRAPH ¥85, 19(3), 31-40, ACM SIGGRAPH New York, 1985

[Cohen88] Cohen, M. F.,Chen, S. E., Wallace, J. R., Greenberg, D. P.: *A Progressive Refinement Approach to Fast Radiosity Image Generation*. In Computer Graphics SIGGRAPH ¥88, 22(4), 75-84, ACM SIGGRAPH New York, 1988

[Cohen93] Cohen, M. F., Wallace, J. R.: *Radiosity and Realistic Image Synthesis.* Academic Press Professional, 1993

[Ferschin94] Ferschin, P., Tastl, I., Purgathofer, W.: A Comparison of Techniques for the Transformation of Radiosity Values to Monitor Colors. First IEEE International Conference on Image Processing, November 13-16, 1994, Austin, Texas, USA.

[Goral84] Goral, C. M., Torrance, K. E., Greenberg, D. P., Battaile, B.: *Modeling the Interaction of Light Between Diffuse Surfaces*. In Computer Graphics SIGGRAPH ¥84, 18(3), 213-222, ACM SIGGRAPH New York, 1984

[Heckbert92] Herkbert, P.:*Discontinuity Meshing for Radiosity*. In Third Eurographics Workshop on Rendering., May 1992, 203-226, Bristol, 1992

[Immel86] Immel, D., S., Cohen, M.,F., Greenberg, D., P.: *A Radiosity Method for Non-Diffuse Enviroments*. In Computer Graphics SIGGRAPH ¥86, 20(4), 133-142, ACM SIGGRAPH New York, 1986

[Kajiya86] Kajiya, J. T.: *The Rendering Equation*. In Computer Graphics SIGGRAPH ¥86, 20(4), 143-150, ACM SIGGRAPH New York, 1986

[Lischinsky92] Lischinsky, D., Tampiere, F., Greenberg, D., P.: *Discontinuity Meshing for Accurate Radiosity*. In IEEE Computer Graphics and Applications,vol. 12 / no. 6,25-39,1992

[Nishita85] Nishita, T., Nakamae, E.: *Continuous tone representation of three-dimensional objects taking account of shadows and interreflection*. In Computer Graphics SIGGRAPH ¥85, 19(3), 23-30, ACM SIGGRAPH New York, 1985

[Shirley90] Shirley, P.: *A Raytracing Method for Illumination Calculation in Diffus-Specular Scenes*. In Proceedings of Graphics Interface ¥90, 1990

[Shirley91] Shirley, P.: *Time Complexity of Monte Carlo Method*. In Proceedings of Eurographics ¥91,North Holland Publishing, 1991

[Sillion89] Sillion, F., Puech, C.: *A general two-pass method integrating specular and diffuse reflection*. In Computer Graphics SIGGRAPH ¥89, 23(3), 334-344, ACM SIGGRAPH New York, 1989

[Sillion91] Sillion, F., Arvo, J., Westin, S., Greenberg, D., P.: *A global illumination solution for general reflectance distributions*. In Computer Graphics SIGGRAPH ¥91, 25(4), 187-196, ACM SIGGRAPH New York, 1991

[Sillion94] Sillion, F., Puech, C.: *Radiosity and Realistic Image Synthesis*. Morgan Kaufman Publishers, 1994

[Vedel91] Vedel, C.: *Improved Storage and Reconstruction of Light Intensities on Surfaces*. In Third Eurographics Workshop on Rendering., May 1992, 113-121, Bristol, 1992

[Wallace87] Wallace, J., R., Cohen, M., F., Greenberg, D., P.: *A Two-Pass Solution to the Rendering Equation: A Synthesis of Ray Tracing and Radiosity Methods*. In Computer Graphics SIGGRAPH ¥87, 21(4), 311-320, ACM SIGGRAPH New York, 1987

[Wallace89] Wallace, J., R., Elmquist, K., A., Haines, E., A.: *A Ray Tracing Algorithm for Progressive Radiosity*. In Computer Graphics SIGGRAPH ¥89, 23(3), 315-324, ACM SIGGRAPH New York, 1989