A graphical modelling technique for time-space simulations

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Abstract

Animation techniques for a dynamic model representation can be conveniently considered to describe time-space evolutions of scientific phenomena. The static representation of a model is obtained by means of Dynamic Spline Curves; starting from such curves, cross-sectional methods are used for the surfaces generation. The integration of the physical laws, describing the time evolution in the mathematical model, allows the simulation of a dynamic behaviour.
The paper describes the procedure we followed for the implementation of a dynamic model, that we named "Dynamic Surface", and shows its application concerning the waves propagation on the water surface.

1 Introduction

The visualization of data, coming from numerical simulations or experiments, plays a fundamental role in several scientific research areas, among which are fluid-dynamics, molecular dynamics, biology, and medicine. Such areas, in fact, strongly ask for the representation of a phenomenon evolution by means of dynamic models. Therefore, the scientists must be provided with both "efficient" graphical representations of data and animation techniques in order to more easily interact with multidimensional data.
A mathematical model, which describes the evolution of a phenomenon, has to be included into the geometrical model which represents it. Finally, on one hand, geometrical representations of mathematical models are required to be usable for different scientific areas, computationally convenient and "close" to the real model. On the other hand, the provided animation techniques have to be efficient for both the description of the phenomenon evolution and the interpretation of details not otherwise understandable from static pictures.

The goal of our work has been focussed towards the definition of a procedure suitable to represent a dynamic model. We named it a "*Dynamic Surface*", that is a geometric model that follows a dynamic behaviour described by means of a physical law. The main gain using the dynamic surface models for a simulation is the possibility to represent a phenomenon starting only from the initial data of the particle system and a physical law that controls the system evolution.

The first section of the paper includes an outline of the mathematical models we used, the Dynamic Spline Curves [8]. Such curves, which follow a motion equation, have been used as surface generators by means of cross-sectional design techniques. In particular, among cross-sectional methods, a 3D skinning method with spine controlled interpolation has been realized in which the spine curve is represented by a dynamic spline curve.

Some applications regarding the propagation of perturbations on the water surface are described in section 3. These examples represent prototype experiences we are carrying out in order to simulate a ship moving with forward velocity in a complex system of waves coming from different directions and taking into account also the effect of motion and the seakeeping of a ship.

Over the years, computer graphics researchers have developed numerous models capable of simulating currents and waves. Such models worked specifically to this goal, by solving equations numerically [7] [12], or by visual simulations [2][10].

We present a simulation of water surface as a possible application of our dynamic model which is suitable for simulating more general dynamic behaviours. Our technique wants to be a good compromise between efficient run-time animations and good realistic effects. A set of primitives for the representation of dynamic surfaces has been actually included in an extended version of the ANIMA environment, that is an interactive system we previously designed for scientific data animation [5][9]. In particular, the reader is reminded that ANIMA (implemented by means of PEX [6]) allows the user for both to visualize two and three-dimensional dynamic models and to define animated sequences for studying time-space evolution phenomena.

2 Dynamic Surfaces

As we have already mentioned, by the term Dynamic Surface we refer to a generic dynamic model, which is a geometric model used to represent the evolution of a phenomenon.

In this section we briefly describe the two techniques used in order to realize these dynamical models: the dynamic spline curves and 3D skinning.

A dynamic spline curve is used as spine curve. A finite number of planar curves is located on a spine curve; the planes of such curves are perpendicular to the tangent vector of the spine curve at some common points. Then, the dynamic surface model is built by skinning techniques.

The **dynamic spline curves**, introduced in [8], are used in order to simulate the dynamics of a trajectory controlled by motion kinematics and by the fields interacting

with the particles. More precisely we suppose to have a sequence of particles on a curve. Each particle moves on a trajectory, and at each time, the location of the particles determines the shape of their interconnecting curve. This curve is generated by a C^2 piecewise interpolation, where each piece is determined by a third degree Hermite polynomial interpolation. The $i-$th piece, defined by the endpoints P_{ib} and P_{ie}, and by the corresponding tangent vectors $\overrightarrow{T}_{ib}$ and $\overrightarrow{T}_{ie}$, is expressed in terms of third degree Hermite polynomial basis, $H_k(t)$, $k = 0, 1, 2, 3$, by :

$$P_i(t) = P_{ib}H_0(t) + P_{ie}H_1(t) + \overrightarrow{T}_{ib}H_2(t) + \overrightarrow{T}_{ie}H_3(t)$$

where $t \in [0, 1]$, and $H_0(t) = 2t^3 - 3t^2 + 1$; $H_1(t) = -2t^3 + 3t^2$; $H_2(t) = t^3 - 2t^2 + t$; $H_3(t) = t^3 - t^2$.

The trajectory evolution is completely determined by dynamically linking together the curve pieces under field control. The term 'dynamic spline' is derived from the fact that the trajectory of the $i-$th piece of curve is affected by fields or forces applied at time step i, $f_1, f_2, .., f_n : R^3 \rightarrow R^3$ at the particle represented by the point P_{ib}.
The new curve piece satisfies the constraints:

$$\begin{cases} P_{ib} = P_{i-1e} \\ \overrightarrow{T}_{ib} = \overrightarrow{T}_{i-1e} \\ \overrightarrow{T}_{ie} = (1 - F)(\overrightarrow{T}_{i-1e} + \sum_k A_k f_k(P_{ib})) \\ P_{ie} = P_{ib} + \frac{1}{2}(\overrightarrow{T}_{ib} + \overrightarrow{T}_{ie}) \end{cases}$$

where $F \in [0, 1]$ allows for the simulation of motion with drag ($F = 0$, represents the special case of friction-less motion), while $A_k \in [0, 1]$ is the field affinity parameter which specifies the sensitivity the particle system has to each field f_k ($A_k = 0$, the particle system ignores the field f_k). Because $\overrightarrow{T}_{ib} = \overrightarrow{T}_{i-1e}$, the C^1 parametric continuity over the whole trajectory is guaranteed, which means that the continuity of velocity is maintained, that is essential to natural motion. Moreover, even if is not essential for our aims the trajectory curve is also C^2 continuous.

In order to easily create a dynamic surface starting from a dynamic spline curve, **skinning techniques** have been used [11]. We should notice that operators on curves to create surfaces for example extrusion, joining, surfaces of revolutions, and ruled surfaces, are well known techniques in geometric modelling [3]. According to these techniques we obtaine a surface by interpolating a set of planar curves, called section curves, positioned and oriented in space with respect to a so-called spine curve which, in our case, is represented by the dynamic spline curve. Each section curve is positioned on the spine curve in such a way that one of its points, chosen by the user, coincides with a given reference point on the spine curve. Moreover, each section is oriented so that its plane is perpendicular to the tangent vector of the spine curve at the reference point. The resulting skinning surface

interpolates the section curves in a parametric direction and the spine curve in the other one, and a C^2 continuity has to be guaranteed.
Note that the section curves are given by the phenomenon geometry, and can be obtained by interpolation of the data defining the problem.
Based on B-splines, the skinning technique can be defined as follows.
Let :

$$C_k(u) = \sum_{i=0}^{n} N_{i,n}(u) P_{i,k} \qquad k = 0, \ldots, K$$

be the section curves in the parametric direction u, where $P_{i,k}$ are the control points and $N_{i,n}$ the normalized B-spline basis functions. We assume that all the $C_k(u)$ curves are defined on the same knot vector U and have common degree. Let :

$$C_s(v) = \sum_{i=0}^{n_s} N_{i,n_s}(v) Q_i$$

be the spine curve, and $\{\widehat{v_k}\}$, $k = 0, .., K$, be the spine parameters corresponding to the reference points on the $C_s(v)$ where the section curves are positioned. For each index i , the control points $P_{i,k}$, $k = 0, .., K$, in the longitudinal direction, are interpolated with the spine curve in order to satisfy the conditions:

$$C_s(\widehat{v_k}) = P_{i,k} \qquad k = 0, .., K.$$

The control polygons of the interpolation curves form a control mesh which defines a B-spline skinning surface:

$$S(u, v) = \sum_{i=0}^{n_s} \sum_{j=0}^{n} N_{i,n_s}(v) N_{j,n}(u) Q_j^{(i)}$$

of degree (n, n_s) and knot vectors U, V, respectively, in u, v parametric directions, where $Q_j^{(i)}$ is the j-th control point of the interpolating curve through $P_{i,0}, .., P_{i,K}$.
If the section curves turn out to be incompatible, that is with different degrees and knot vectors, knot-insertion and degree elevation techniques must be used, as suggested in [11].
The user interactively chooses a point on the section curve and defines the law of motion that the particle system on the spine curve must follow. For any animation time-step the Frenet frame [3] corresponding to each reference point on the spine curve has to be determined and suitable coordinate transformations are applied to the control points of the section curves in order to locate the section curves, as previously described.
The motion law is applied to the points of the dynamic spline curve, which is the spine curve. Then, the positioned and oriented section curves are interpolated and, finally, the dynamic surface, obtained by skinning technique, is visualized.

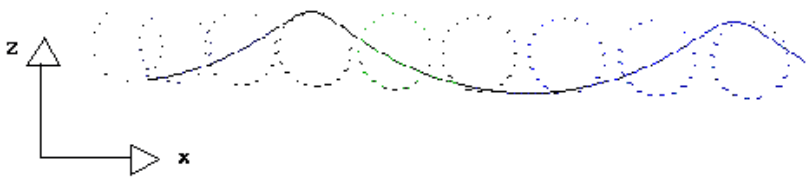

Figure 1: Dynamic spline and trajectories described by water particles

3 An application of dynamic surfaces: wave propagation on water surfaces

The dynamic surface model has been applied to the study of some wave phenomena or perturbations on a medium. If we want to build a graphical model of waves as close as possible to the "real world", we need a mathematical model for the medium which, at the beginning, represents the particle system ordinarily at rest, and, then, could integrate the behaviour of the wave that invests the particle system.
In particular, the models realized allowed for the simulation of sinusoidal plane waves propagating on the water surface.
The wave profile has been reproduced by means of a dynamic spline curve and a dynamic surface has been used to represent the water surface.
Considering the Lagrangian method to model the fluid motion, each particle (i.e. each point) on the water surface describes a circular trajectory around its rest position (x_0, y_0, z_0) [4]. If we assume that the XY plane is the plane of the water at rest, the equation of the particle motion is then:

$$\begin{cases} x = x_0 + r\sin(kx_0 - \omega t) \\ z = z_0 - r\cos(kx_0 - \omega t) \end{cases} \tag{1}$$

where k is the wave number, r is the radius of orbit, and ω is the angular speed. We denote the phase speed by $c = \frac{\omega}{k}$ and the phase angle by $\phi = kx_0 - \omega t$.
The previous equations for a given t represent the perturbation or displacement from a reference position of the particles which lie on the straight line where the wave is propagated at the instant t (see Figure 1).
In order to represent the perturbation evolution into a particular area for successive instants of time, we can determine the dynamic surface as a function of time, and, then, generate successive frames of animation that let us observe the motion of the water surface, that is the effect of the propagation of a perturbation over a given interval of time.

Note that force fields are included in the terms k and ω which determine the wave speed.
In the remainder of this section we will consider the following phenomena : wave trains

that overlap, the effect of the wind on the top of the crests, and the effect of the water depth.

- Overlapping of wave trains

We consider the motion resulting from waves which are propagating in the same direction. A wave train is a rectangular box on the water surface containing waves of the same basic characteristics (height, period, and wavelength) and of the same phase origin.

The height and the wavelength of waves within a train are made a function of time, so that the lifetime of the train is taken into account. By default, the direction of motion for a wave train is towards the positive X axis. In accordance with the overlapping principle we can determine the position of a point on a surface as the sum of the displacements from the point's rest position due to all active trains covering this point.
In Figure 2 the water surfaces of two wave trains which have different characteristics are illustrated. The first train has an initial height greater than the second one that diminishes as function of time, while the second train has a height which does not change.
Moreover, the two groups have different wavelengths and periods. Finally, from Figure 3 we can observe 5 time steps of animation considering the resulting perturbation due to both the wave trains.

- Effect of wind on the top of the wave crest

In order to represent the effect of wind on the top of the wave crest, so called *kludge* phenomenon, equations (1) are changed as follows [4]:

$$\begin{cases} x = x_0 + r\sin(kx_0 - \omega t - \beta\Delta z\Delta t) \\ z = z_0 - r\cos(kx_0 - \omega t - \beta\Delta z\Delta t) \end{cases}$$

where the quantity $\beta\Delta z\Delta t$ has been subtracted to the phase angle ϕ, β is the coefficient of proportionality, $\Delta z = z - z_0$ and Δt is included because this effect is proportional to time. This modification causes the point to accelerate at the top and decelerate at the bottom. This effect can be observed in Figure 4 for 5 animation time steps; in the example we set $\omega = \pi/10$, $r = 0.05$, $k = 14$ and $\beta = 30.5$.
In this example the external dynamic effect of the wind is implicitly considered in the above equations through the term $\beta\Delta z\Delta t$.

- Effect of water depth

We assume the depth h of the water, measured positively down from the water level at rest, diminishes for increasing values of x_0 according to the law :

$$h = \frac{1}{x_0^3 + 1}. \tag{2}$$

The effect of the depth is to alter the wavelength and, consequently, the phase speed; we can assume that the period is not affected. If we call k_∞ the wave number at infinite depth, a good approximation for the wave number k at depth h is [4]:

$$k\tanh(kh) = k_\infty. \tag{3}$$

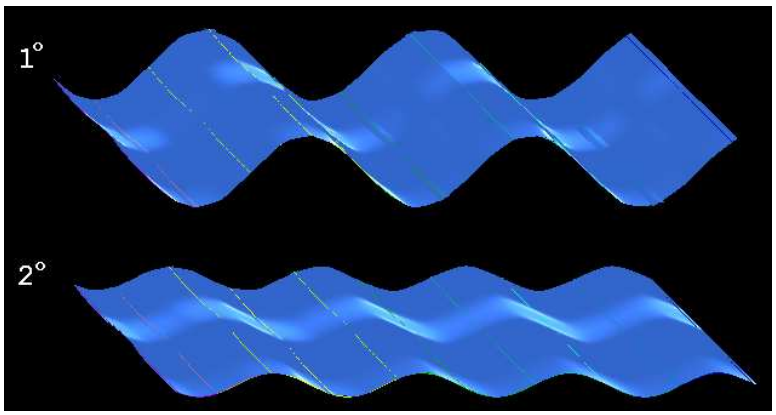

Figure 2: Water surface for two different wave trains

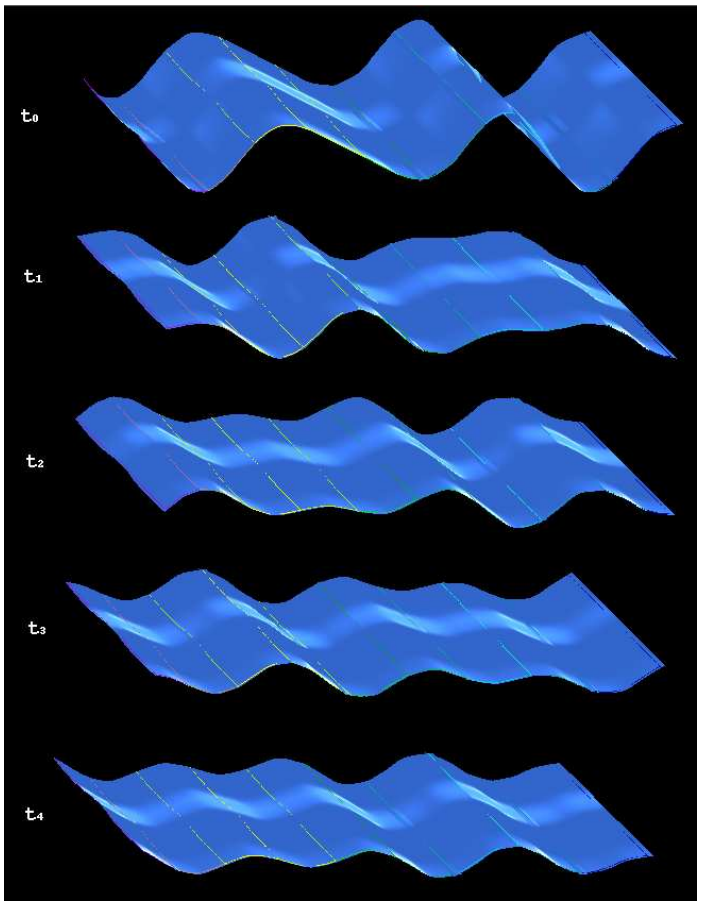

Figure 3: Overlapping of two different wave trains

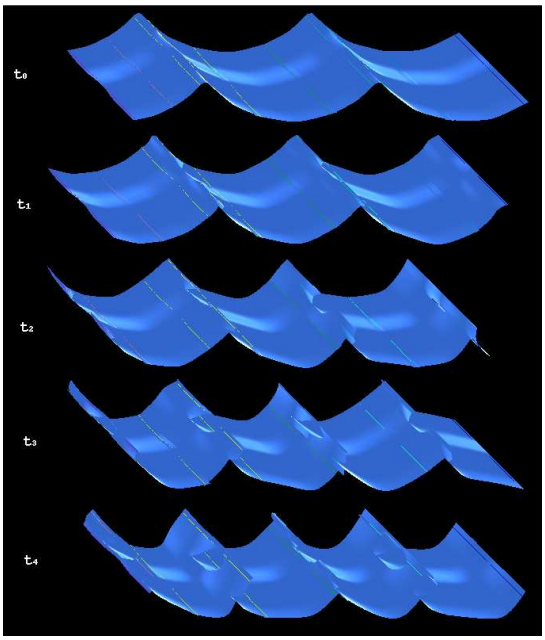

Figure 4: Effect of wind

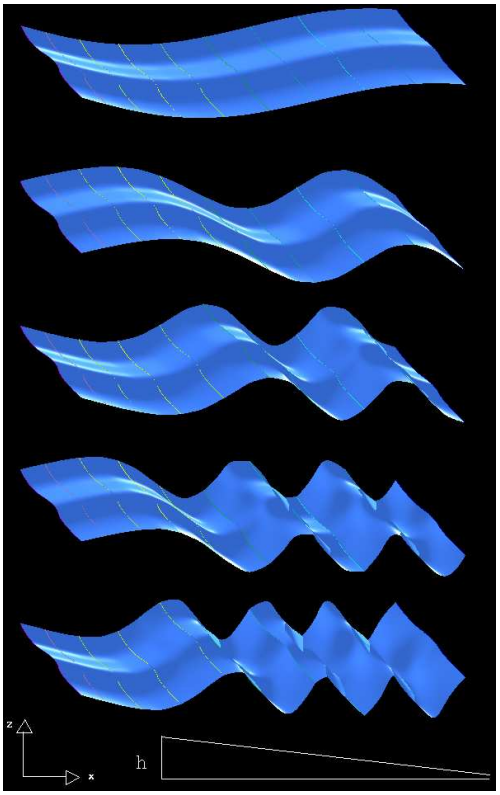

Figure 5: Effect of water depth: h diminishes for increasing x

At small depth we can consider the following approximation for the equation (3): $k = \frac{k_\infty}{\sqrt{\tanh(k_\infty h)}}$.

Furthermore, the waves are refracted because of the phase speed is affected. To compute the angles the wave fronts make at depth h with respect to those at infinite depth, we consider:

$$\frac{\sin(\alpha_h)}{\sin(\alpha_\infty)} = \frac{c_h}{c_\infty}.$$

This angle in our dynamic spline model will affect the orientation of the spine curve with respect to the X axis, when the wave progress is assumed in the positive X direction.
As the depth h diminishes, both the wavelength and the phase speed diminish. Also here the force field is represented through the variation of the speed due to the reduction of h. In Figure 5, 5 frames are shown which illustrate the passage from high depth zone (left) to low depth zone (right), according to equation (2).

4 Conclusions

We described a representation technique for dynamic models that we named "Dynamic Surface" and we shown some applications for studying plane waves of sinusoidal shape propagating on the water surface. The water surface has been modelled as a parametric dynamic surface further animated by including time evolution laws into the mathematical model.
Dynamic Surfaces modelling, included in the ANIMA environment, that is a tool oriented to scientific data animation, is now allowing us to represent motions of more than one object or model in a scene. In particular we are studying, by means of dynamic surface, how to represent the motion of an object relative to another one and visualize, for instance, the effects of the waves propagation on the motion of a boat with forward velocity.

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