Adaptive subpixel cross-correlation in a point correspondence problem

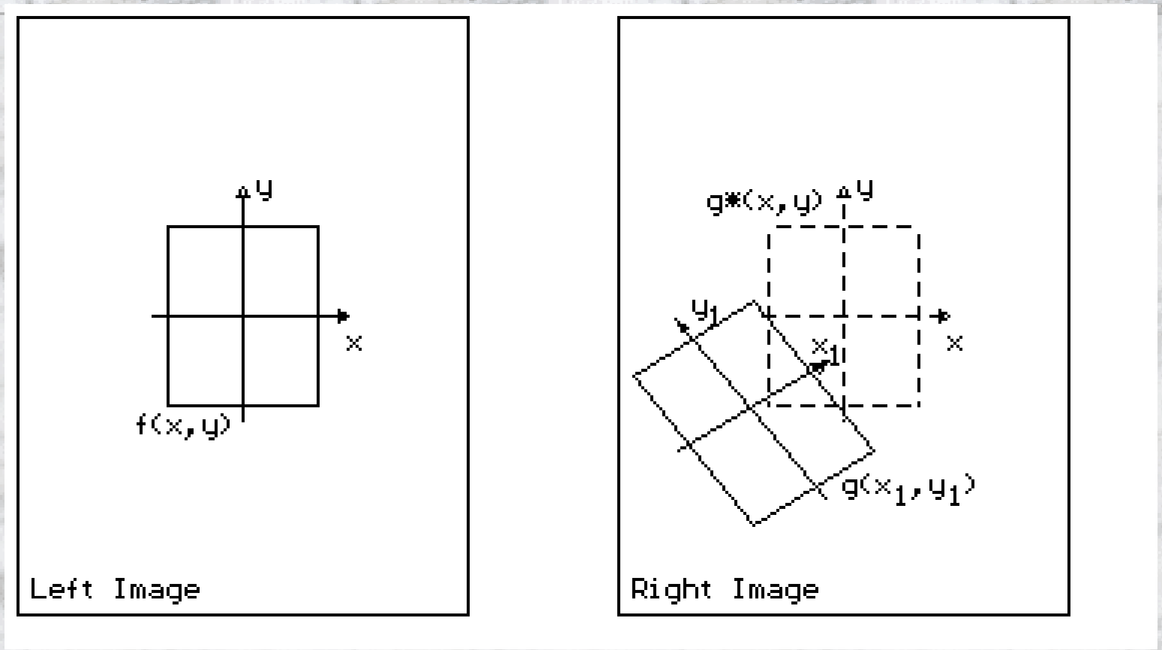
- **The goal of work:** Consecutive application of normalized cross-correlation for subpixel point matching and comparison with least-squares correlation.

Low-level image matching basic approaches

Least-Square Correlation

Cross-correlation

Subpixel cross-correlation approach



Transformation between left and right windows:

$$x_1 = a_1 + a_2 x + a_3 y$$

$$y_1 = b_1 + b_2 x + b_3 y$$

p = (*a1*,*a2*,*a3*,*b1*,*b2*,*b3*)*T* - vector of affine transformation parameters

Correlation coefficient

$$k(\mathbf{p}) = \frac{\sum_{(x,y)} f(x,y)g(x_1,y_1) - N\bar{f}\bar{g}}{(\sum_{(x,y)} f^2(x,y) - N\bar{f}^2)^{1/2}(\sum_{(x,y)} g^2(x_1,y_1) - N\bar{g}^2)^{1/2}}$$

Linearization step

$$k'(\Delta\mathbf{p}) = \frac{\Delta\mathbf{p}^T(\sum_{(x,y)} f\mathbf{g} - N\bar{f}\bar{\mathbf{g}})(\sum_{(x,y)} f\mathbf{g}^T - N\bar{f}\bar{\mathbf{g}}^T)\Delta\mathbf{p}}{\Delta\mathbf{p}^T(\sum_{(x,y)} \mathbf{g}\mathbf{g}^T - N\bar{\mathbf{g}}\bar{\mathbf{g}}^T)\Delta\mathbf{p}} = \frac{\Delta\mathbf{p}^T\mathbf{A}\Delta\mathbf{p}}{\Delta\mathbf{p}^T\mathbf{B}\Delta\mathbf{p}} \to \max$$

Generalized eigenvalue problem:

$$\mathbf{A}\,\Delta\,\mathbf{p} = \lambda\,\mathbf{B}\,\Delta\,\mathbf{p}$$

Theorem

Consider any vector **a** of dimension *n* and symmetric, positively defined and non-singular matrix **B** of dimensions *n* x *n*.
Then for solutions of a generalized eigenvalues problem

$$\mathbf{Ax} = \lambda \mathbf{Bx}$$

where

$$\mathbf{A} = \mathbf{a}\mathbf{a}^T$$

the following statements are valid:

1) There are two generalized eigenvalues: $\lambda_1 = 0$ of *n-1* fold and $\lambda_2 > 0$ of *1* fold;
2) Generalized eigenvector corresponding to *2* is given by the formula

$$\mathbf{x} = \mathbf{B}^{-1}\mathbf{a}$$

3) $\lambda_2 = \mathbf{a}^T\mathbf{x}$, where **x** - eigenvector corresponding to λ_2.