The Comparison of Illumination Maps Technique in Computer Graphics Software

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Abstract

This paper is a continuation of our previous work [1] on the comparison of computer graphics software claiming to physically accurate global lighting simulation. Physical accuracy is the main criterion for this comparison. This work devotes to illumination maps technique. New tests and comparative methods are proposed.

Keywords: *GraphiCon'98, graphics software, illumination maps, illumination analysis, simulation accuracy, forward Monte Carlo ray tracing, radiosity.*

1. INTRODUCTION

Illumination maps is a technique that stores results of global illumination analysis (that is distribution of diffuse illuminance in the whole scene) in view-independent way. These data (called "illumination maps") allow us to generate a series of high-quality images (differing by observer position and viewing parameters) having done time consuming global illumination analysis only once. Also they offer real-time walk-through with account for global illumination results with OpenGL based hardware.

In this paper we investigate possibilities to create illumination maps in computer graphics systems used in previous comparison [1]: Lightscape Visualization System (LVS), ver. 3.1.1, distributed by Lightscape Technologies, Inc., and Inspirer system, ver. 5.30, distributed by Integra, Inc., and Radiance ver. 3.0, a public domain system developed by G.Ward.

LVS and Inspirer store the computer illumination directly on the surfaces in the scene. In order to represent variations of illumination across a surface, LVS and Inspirer automatically breaks down the surface into smaller pieces, called elements. The simulation then computes the illumination from a light source to each of the vertices of each element. The set of all the elements and vertices is illumination maps. Also LVS and Inspirer are able to convert created illumination maps to VRML file, that can be navigated for walk-through by corresponding browsers. Radiance in fact does not support illumination maps technique but it has some analog to store the collection of samples computed during one rendering, that can be reused to speedup further renderings possibly with different camera parameters. As far as only illumination maps technique is concerned in this chapter, Radiance is not considered here.

Although LVS and Inspirer have similar mechanism to store illumination at element vertices that allows the user to quickly generate multiple views from the same data, the technology used to create illumination maps is different for each system. The lighting simulation software used in LVS is based on a technology called radiosity; Inspirer offers forward Monte Carlo lighting simulator. The both processes are controllable by a set of parameters and have some automatic tools in order to maintain as efficient a solution as possible. We apply LVS' and Inspirer's methods for following datasets:

- CUBE test, a scene with analytic solution for diffuse-diffuse energy transfer;
- SPHERE test, a scene with analytic solution for diffuse-specular-diffuse energy transfer;
- a practical scene with realistic interior.

All calculations were performed on PC whith Pentium 200MHz processor, 64Mb RAM. All presented images with original quality are available in HTML version of the paper available from:

http://rmp.kiam1.rssi.ru/articles/pals1/lvs_tbt

2. ANALYTIC TESTS

The section describes analytic tests proposed for the comparison and experimental results.

2.1 Cube

This test was proposed by us in [1] to evaluate accuracy of diffuse-diffuse energy transfer.

The first scene used for comparison is simply an interior of a cube (10×10×10 m) with one point light source at its center (Fig.1). Luminous intensity of the light source is equal to 50000 cd that create illumination level 2000 lux at nearest point on wall. The wall material is white with diffuse reflectivity 2/3 so that indirect component takes a great part in whole illumination.

Because of symmetry all cube faces are equivalent. We have chosen several points on cube face where theoretical results are compared with results produced by the systems.

Although the scene is very simple, exact analytical solution for illuminance distribution in it seems to be impossible. We used a combination of Monte Carlo simulation and direct numerical integration of the rendering equation to get accurate estimates of illuminance in chosen points.

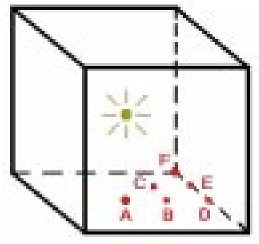


Fig.1. Empty cube.

<u>Reference data</u>

We analyzed luminance in several points on cube face. These points constitute uniform 5×5 grid on the whole face. Due to symmetry there are only 6 different points (they are marked A-F in Fig. 1); in below table we present them in local coordinates system: point (0, 0) is face center, face edges have one ±1 coordinate. Below "semi-theoretical" luminance values for these points are presented. They are computed in the following way. At first we ran Monte Carlo simulation to get luminance distribution in cube faces. Then illuminance of each point was computed by direct integration of energy transfer equation. This equation is equivalent to rendering equation [2], for our case it can be written as:

$$Illum(p) = \int_{\Omega} \frac{Lum(q) \cdot \cos(\alpha) \cdot \cos(\beta)}{r^2(p,q)} dA(q)$$

where

$Illum(p)$ is unknown illuminance in some point p;
$Lum(q)$ is luminance at point q;
Ω integral spans over all surfaces in a scene;
α, β are angles between surface normal at points p and q and segment connecting points p and q;
$r(p,q)$ is Euclidean distance between p and q;
$dA(q)$ is differential area element at point q.

A special trick was used to avoid singularity of this integral at points lying on cube edges (last 3 points in the Table 1).

Point	coordinate (x, y)	luminance (cd/m²)	comment	weight
A	(0, 0)	892.8	face center	1/25
B	(0.5, 0)	768.7		4/25
C	(0.5, 0.5)	686.6		4/25
D	(1, 0)	565.1	middle of edge	4/25
E	(1, 0.5)	522.4		8/25
F	(1, 1)	388.4	cube corner	4/25

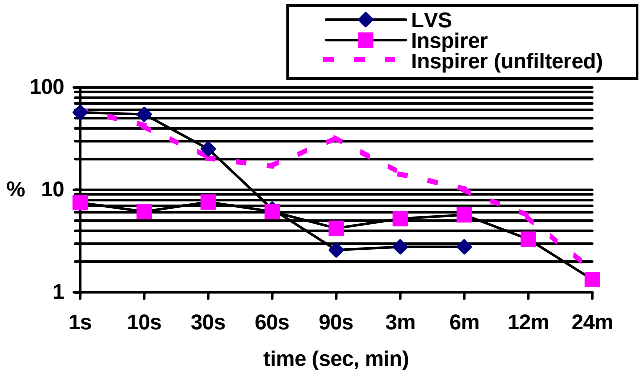
Table 1. Theoretical luminance values for cube.

<u>Experimental results</u>

This test was already used for comparison of the systems in our previous work in [1]. Now we compare newer versions of the systems, and also we have changed the accuracy measure. It is because we have found a drawback in our previous method of simulation accuracy estimate for this scene. The reason for the drawback is that we compared simulation results in each point separately. The result in one point is expressed by a scalar value, and as a result, if a simulation system gradually changes its estimate of this value from, say, underestimate to overestimate, then at some intermediate step the estimate may happen to be very close to the theoretical value (that will be mistakenly recognized as an indication of high accuracy of that system) while continuation of simulation gets worse results. Such situation actually occurs in previous report for Radiance rendering in one point.

To avoid such possibility it is necessary to not use difference of scalar values as an accuracy measure. Now we use instead distance between some distributions, involving many points. In such case an occasional coincidence becomes rather unlike. So, it is better to combine differences for all 6 points for which theoretical values are computed in previous report into a single error estimate (say, we can use the RMS average of the 6 differences). Each point has some weight corresponding to the count of points of the same type on cube face.

All calculations were performed with most accurate settings (Higher Quality Wizard setting was used for LVS, fine geometry mesh subdivision for Inspirer). The relative values are adduced on the Graph 1:



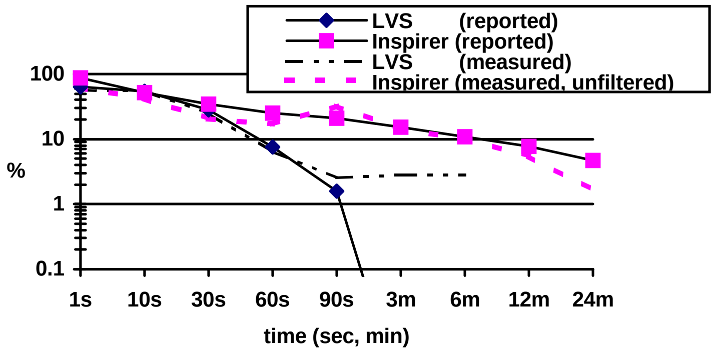
Graph 1. CUBE. Measured error vs. time.

This graph has large enough relative range on Y, so it is presented in logarithmic Y scale. We used two Inspirer's modes in this test: with filtering of illumination maps and without filtering (referred as "unfiltered"). In our assumption Inspirer's filtering may improve accuracy at initial stages while finally (when non-filtered data become accurate enough) it should switch off automatically. This assumption is confirmed by results.

The results show that LVS achieves accuracy error in 2.8% for this scene 10 times faster than Inspirer. It demonstrates undoubted progress with simulation accuracy in new LVS version. In the same time accuracy of the simulation is still limited - LVS stops the simulation on 6-th minute with 2.8%. Another drawback of LVS is lack of accuracy improvement for last 3/4 period of the work.

The advantage of Inspirer is lack of algorithm-specific variables for simulation control and accuracy control that manages the simulation until specified accuracy level will be achieved.

The both systems report progress of the calculation. The meaning of progress reports is quite different: Inspirer reports an estimation of simulation error [1]. LVS reports only a number of initial energy that has been distributed. We compare these reports on Graph 2:



Graph 2. CUBE. Reported error vs. time.

Accuracy estimation in Inspirer statistically predicts measured error of its unfiltered results generated by forward Monte Carlo simulator. LVS' report also reflects current accuracy of LVS radiosity solution process but it reaches 100% of initial energy distribution (i.e. 0% error in that term) on half of total calculation time. The next half of calculations is performed by LVS without any progress report.

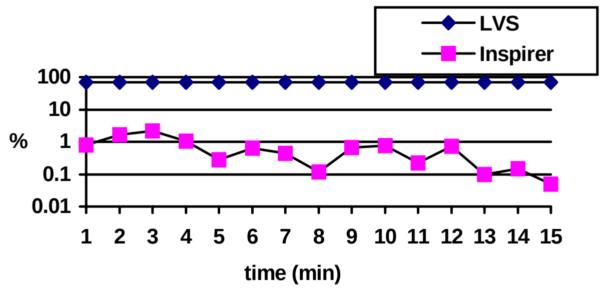
2.2 Sphere

This test is based on observation that ambient illuminance can be exactly calculated under condition that form-factor is the same for all pairs of points. The scene is a diffuse sphere bisected by three mirror coordinate planes. So the volume under consideration consists of 1/8 of the whole ball. This test is designed to check diffuse-specular-diffuse energy transfer.

LVS uses the specular characteristics of a material only when ray tracing. So we can not expect accurate simulation of this scene by LVS. The graph below is not actually a comparison, but simply an illustration of this limitation.

Let us consider 1/8-th part of diffuse unit sphere (produced by its section along coordinate planes) which is closed by three mirror-like circular sectors. In this case, generalized form-factor which characterizes diffuse-diffuse and diffuse-specular-diffuse energy interchange is the same for all pair of points belonging to this part of the sphere. The analytic solution of total illuminance in the center of sphere octant was found by Khodulev A. and Ignatov V. in 1993 (see detailed scene description and derivation of results in Appendix A). The resulted illuminance in that point is 1353.247 lux.

The result of simulation is presented on Graph 3:



Graph 3. SPHERE. Simulation error vs. time.

So diffuse-specular-diffuse energy transfer is correctly simulated only by Inspirer.

3. PRACTICAL TEST

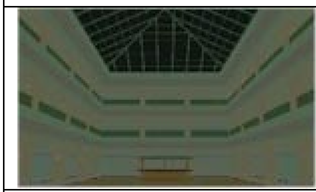
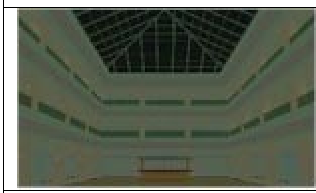
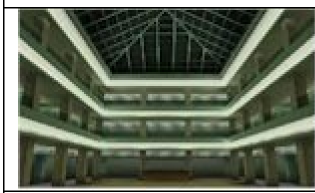
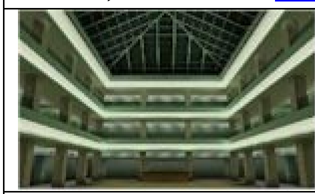
A realistic interior was simulated by LVS and Inspirer and a series of results is displayed using fast OpenGL rendering. This interior contains about 10 thousands of triangles and 2626 luminaries for 3 types of photometric data. The generated images collected in table of images (Table 2) gives us the exhaustive idea about progressive refinement of results. This table also contains statistic information reported during LVS' radiosity and Inspirer's forward Monte Carlo simulation.

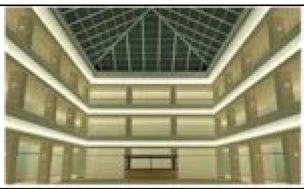
Unfortunately we have neither analytical solution nor a photograph of real interior for this model. Our comparison is based on visual estimation of the realism and the speed of progressive refinement, quantitative comparison of the speed in accordance to reported statistic, and analysis of convergence speed.

3.1 Visual estimation of the realism and the speed of progressive refinement

Table 2 presents a series of images for visual comparison. Images are generated by the two systems in the same time points. The first point corresponds to 1 minute of calculation in batch mode (the time for OpenGL-rendering is not taken into account here), the last one after 2^{12} minutes (i.e. almost 68.5 hours). Each time interval is 2 times longer than the preceding one. We can not run LVS until its "natural" stop after 100% energy shot because it took too much time.

We used default settings with available approximation of results. LVS approximation is ambient lighting to compensate unshot energy. Ambient lighting ratio was set equal to unshot energy reported. Inspirer offers automatic filtration of illumination maps to balance inaccurate illumination of some triangles mainly on early calculation increments.

LVS 3.1.1 (Radiosity processor)	**Inspirer 5.30** (forward Monte Carlo simulator)
	
1 min; **LVS**: 99.52% left[1]; **Inspirer**: 25.73% left[2]	
	
2 min; **LVS**: 99.02% left[1]; **Inspirer**: 17.59% left[2]	
	
4 min; **LVS**: 97.82% left[1]; **Inspirer**: 12.27% left[2]	
	
8 min; **LVS**: 93.93% left[1]; **Inspirer**: 8.74% left[2]	
	
16 min; **LVS**: 87.26% left[1]; **Inspirer**: 6.31% left[2]	
	
32 min; **LVS**: 74.91% left[1]; **Inspirer**: 4.34% left[2]	
	
64 min; **LVS**: 57.8% left[1]; **Inspirer**: 3.06% left[2]	
	
128 min; **LVS**: 46.24% left[1]; **Inspirer**: 2.19% left[2]	
	
256 min; **LVS**: 43.05% left[1]; **Inspirer**: 1.60% left[2]	
	
512 min; **LVS**: 37.71% left[1]; **Inspirer**: 1.16% left[2]	
	
1024 min; **LVS**: 33.44% left[1]; **Inspirer**: 0.82% left[2]	

2048 min; **LVS**: 30.8% left[1]; **Inspirer**: 0.58% left[2]

4096 min; **LVS**: 28.96% left[1]; **Inspirer**: 0.41% left[2]

- [1] - LVS produces progress reports in terms of initial energy. For example, 33.44% left means that 33.44% of initial energy still unshot. The actual accuracy level is unknown.
- [2] - Inspirer reports progress in form of error estimation

The series of images gives the idea of progressive refinement of results in the two systems. Progressive refinement feature means that the user is provided with the whole image from very beginning and this image is gradually upgraded as a whole. LVS does not upgrade an image as a whole, its output is something like block by block rendering over ambient illumination. Moreover, the comparison analysis of image convergence in next section shows that series of LVS images diverge during first hour of run. Naturally, the initial image produced by Inspirer appears rough, but it shows complete lighting distribution in the scene. After a lapse of time, as new information pertaining to the image is being collected from the lighting analysis, the image changes as a whole and rapidly converges to final result.

The approximation algorithm implemented in Inspirer seems to be more powerful than LVS' technique of ambient lighting. Lighting simulation is typically time consuming process, but Inspirer is capable to produce rough but complete and acceptable illumination results at the interactive speed for practical scenes.

Looking at illumination maps we see that LVS produces something like shadows near columns:

LVS' fragment

Inspirer's fragment

It is rather a drawback in LVS because there should be no such shadows due to mutual diffuse interreflections and big number of light sources located close to each other in the scene. The probable reason is incomplete calculation at this step.

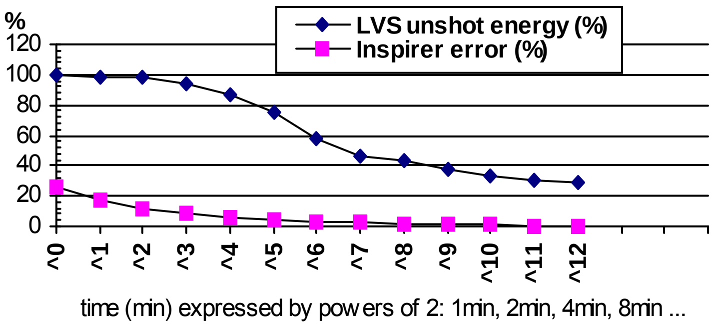
Another impression that concerns accuracy is color reproduction was discussed in [1] and still actual for this test. The images of both systems look quite different in color and contrast. The difference is also noticeable for LVS' images on different stages. It looks as lack of energy balance even after ambient approximation of unshot energy. Another possible reason is the difference in schemes of conversion from physical luminance units to RGB triplets sent to display and/or difference in color spaces. Inspirer allows the user to control the conversion from physical luminance units to RGB and to adjust color space to monitor profile. Inspirer's images above were received for Barco Mega Calibrator monitor profile. LVS has only brightness and contrast controls. LVS' images were received for maximal settings of brightness and contrast from recommended range to provide better compliance with Inspirer's images.

3.2 Quantitative comparison of the speed

The reported progress meaning is quite different for LVS and Inspirer. Inspirer reports an estimation of simulation error [1]. Accuracy measure is not available in LVS. It reports only a number of initial energy that has been distributed.

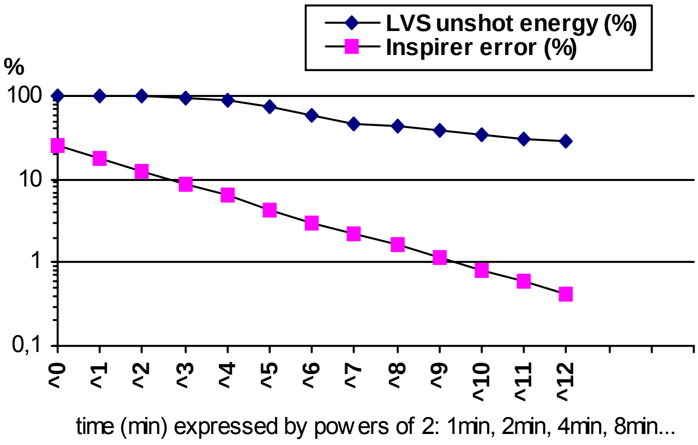
The stop of simulation in Inspirer coincides with the achievement of desired accuracy level. The whole simulation time is subdivided on increments of specified time length. The current accuracy level achieved is monitored for each increment. LVS reports the progress in term of initial energy continuously until 100% of initial energy will be distributed. Then it probably will continue to work in quiet mode without any numeric monitoring as it was observed for CUBE test.

The reported progress is single available to the user numeric information during simulation. It looks as following:



Graph 4. Reported progress vs. time.

A unique feature of Inspirer accuracy control is a time prediction of the completion of Monte Carlo simulation with specified accuracy level. It is based on the fact that a precision of Monte Carlo simulation result is statistically improved as square root of time. This fact is reflected on the graph in logarithmic scale (Graph 5): logarithm of error is changed linearly with logarithm of time.



Graph 5. Reported progress vs. time

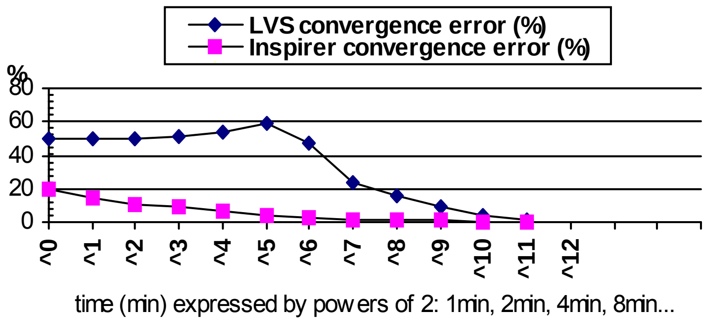
3.3 Analysis of convergence speed

To perform the quantitative comparison we compare convergence speeds for the two systems. This comparison is based on calculation of a distance between intermediate images and some reference image. The reference image for each system is a image calculated for reasonable long time. In our case this time equals to 2^{12} minutes (about 68.5 hours). The distance between two images is the relative form of ordinary L^2 distance in RGB frame buffer space:

$$d(f,g) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(f_i - g_i)^2}$$

$$d_{rel}(f,g) = \frac{d(f,g)}{\|f\|}$$

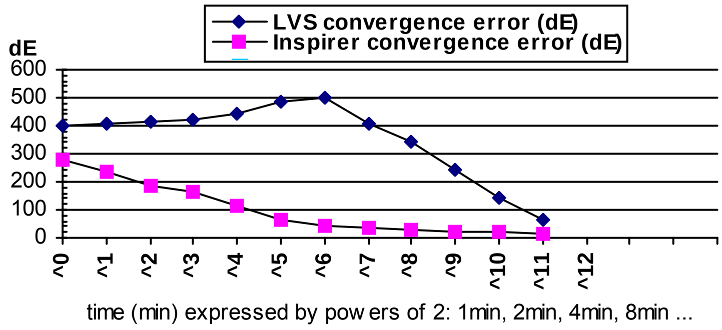
where N is number of pixels; f is considered as a color component of reference image, g - as corresponding color component of compared with..The resulted distance is average for all RGB components. The results are presented on the graph below:



Graph 5. Convergence of images to reference one vs. time. (L^2 metric in RGB)

Note: the convergence results are different for each system; its accuracy is not analyzed here as we have not analytical solution for this model.

Also we can compare image convergence in objective manner as a Euclid difference in CIE Lab (1976) system that is simplified mathematical approximation to a uniform color space composed of perceived color differences [3].

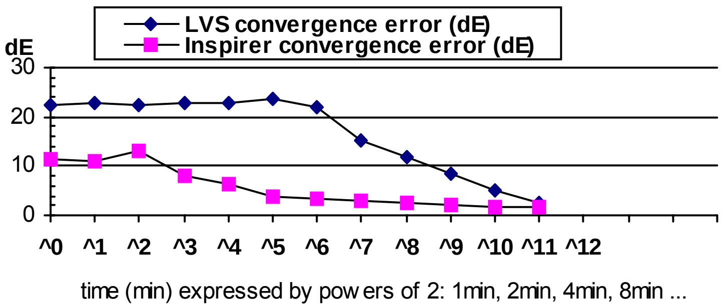


Graph 6. Convergence of images to reference one vs. time. (Euclid metric in Lab CIE 1976)

Note: Unfortunately LVS does not report what monitor profile parameters are used for image synthesis, so we convert the images from RGB frame buffer space to Lab space for both systems with Barco Mega Calibrator monitor profile, which is used by default in Inspirer.

As can be seen from the above graph, LVS' results diverge during first hour of run. So we should conclude again that Inspirer's progressive refinement looks much more attractive than LVS' one.

Finally, we compare the convergence with improved color difference formula recommended by the CIE Technical Committee 1-29 in 1994 [4].



Graph 7. Convergence of images to reference one vs. time. (Euclid metric in Lab CIE 1995)

Here we see Inspirer's divergence for 4 minute. LVS' results diverge here during first half of hour

4. CONCLUSION

LVS produces acceptable result for simple diffuse-diffuse test 10 times faster than Inspirer but failed to improve achieved accuracy level. The relation between LVS and Inspirer speeds is changed to opposite for complex practical scenes with a lot of light sources. Such important features as progressive refinement of the result, accurate color reproduction, accuracy control and completeness of global illumination model are much more attractive in Inspirer.

5. ACKNOWLEDGMENTS

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6. REFERENCES

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Аннотация

Сравнение методов расчета сеток освещенности в графических приложениях.

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Данная статья является продолжением нашей предыдущей работы [1] по сравнению графических приложений претендующих на физически точное моделирование глобальной освещенности. В работе рассматриваются компоненты систем связанные с расчетом сеток освещенности при этом за основной критерий оценивания принята точность моделирования. Предложены новые тесты и методы сравнения.

Appendix A

As first step in our theoretical analysis we consider the complete diffuse sphere without any mirrors inside. Such test was not actually used in the current work. Let

x, y denote arbitrary points on the sphere S;

$k(x)$ is diffuse reflectivity at point x;

$L(x), I(x)$ luminance and illuminance at point x on the sphere surface;

$I_d(x)$ is direct illuminance at point x;

$\int_S f(x)dx$ means integral of a function f(x) over the whole sphere S (so dx is two-dimensional differential);

Then the basic energy equilibrium equation can be written as:

$$L(x) = \frac{k(x)}{\pi} \cdot I(x)\,; \tag{1}$$

$$I(x) = \int_S F(x,y)L(y)dy + I_d(x)\,, \tag{2}$$

where $F(x,y)$ is the form factor between two differential elements at points x and y:

$$F(x,y) = \frac{\cos(\alpha)\cdot\cos(\beta)}{r^2}\,,$$

where α, β are the two incident angles at points x and y and r is the distance between these points.

The key consideration that enables exact calculation here is that all form factors for sphere are equal. Indeed, let us consider two points x and y with angular distance between them a. Then

$r = 2\cdot\sin(\frac{a}{2})$, and $\alpha = \beta = \frac{\pi}{2} - \frac{a}{2}$. Thus

$$F(x,y) = \frac{\cos(\frac{\pi}{2} - \frac{a}{2})^2}{(2\cdot\sin(\frac{a}{2}))^2} = \frac{1}{4}\,.$$

It means that the integral in (2) is constant (does not depend on x):

$$I_a = \int_S F(x,y)\cdot L(y)dy = \frac{1}{4}\int_S L(y)dy \tag{3}$$

and

$$I(x) = I_a + I_d(x)\,. \tag{4}$$

To compute the ambient part I_a substitute (1) and (4) into (3):

$$I_a = \frac{1}{4\cdot\pi}\int_S k(y)I(y)dy =$$

$$\frac{1}{4\cdot\pi}\int_S (k(y)\cdot I_a + k(y)I_d(y))dy =$$

$$\frac{I_a}{4\cdot\pi}\int_S k(y)dy + \frac{1}{4\cdot\pi}\int_S k(y)I_d(y)dy .$$

So

$$I_a\cdot\left(1-\frac{1}{4\cdot\pi}\int_S k(y)dy\right) = \frac{1}{4\cdot\pi}\int_S k(y)I_d(y)dy ,$$

and finally

$$I_a = \frac{1}{4\cdot\pi}\frac{\int_S k(y)I_d(y)dy}{1-K} , \qquad (5)$$

where

$$K = \frac{1}{4\cdot\pi}\int_S k(y)dy \qquad (6)$$

is the average reflectivity.

The formulae (1), (4)-(6) allow us to compute exact luminance value $L(x)$ for any direct illumination distribution $I(y)$ and reflectivity distribution $k(y)$.

Now let us proceed with analysis of our test - 1/8 of the sphere. Let us consider S_8 - 1/8th part of diffuse unit sphere S (produced by its section along coordinate planes) which is closed by three mirror-like circular sectors. In this case, generalized form-factor which characterizes diffuse-diffuse and diffuse-specular-diffuse energy interchange is the same for all pair of points belonging to S_8. Therefore ambient illuminance can be calculated as follows:

$$I_a = \int_{S_8} F(x,y)\cdot L(y)dy .$$

Form factor $F(x,y)$ between two differential elements at points x and y is the constant:

$$F(x,y) = F =$$

$$\frac{1}{4}(1+k_a+k_b+k_c+k_{ab}+k_{ac}+k_{bc}+k_{abc});$$

$$L(x) = \frac{k}{\pi}\cdot I(x) = \frac{k}{\pi}\cdot(I_a + I_d(x)) ;$$

$$I_d(x) = D_0(x) + D_a(x)\cdot k_a + D_b(x)\cdot k_b +$$

$$D_c(x)\cdot k_c + D_{ab}(x)\cdot k_{ab} + D_{ac}(x)\cdot k_{ac} +$$

$$D_{bc}(x)\cdot k_{bc} + D_{abc}(x)\cdot k_{abc}$$

where:

k is diffuse reflectivity of S_8 surface;

k_x is coefficient of specular reflection from mirror which is perpendicular to x axis;

$k_{xy} = k_x\cdot k_y$;

$k_{xyz} = k_x\cdot k_y\cdot k_z$;

$D_{xyz}(x)$ is illumination produced by the virtual light which represents real light reflected over planes x, y and z.

Repeating calculation from above for I_a, we find:

$$I_a = \frac{k\cdot F}{\pi}\int_{S_8}(I_d(y)+I_a)dy .$$

Let $\hat{k} = \frac{k\cdot F}{\pi}$, then:

$$I_a\cdot(1-\hat{k}\cdot Area(S_8)) = \hat{k}\cdot\int_{S_8} I_d(y)dy ;$$

$$I_a = \frac{\hat{k}\cdot\int_{S_8} I_d(y)dy}{1-\hat{k}\cdot Area(S_8)} . \qquad (7)$$

The test data represents 1/8th part of diffuse unit sphere with diffuse reflectivity k=0.7 closed from all sides by mirror-like circular sectors with 1.0, 0.8, and 0.6 coefficient values of specular reflection for XY-, XZ-, and YZ- planes accordingly. The sphere octant is illuminated by one uniform point light source located at point <0.2, 0.4, 0.6> and having intensity 100 candelas. The sphere-like shape is produced by triangulization with fine mesh. Direct and ambient illuminance is calculated at the point $<\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}>$ lying on the sphere:

$$I_d = 686.598 ;$$

$$I_a = 666.649 ;$$

$$I = I_d + I_a = 1353.247 \text{ (lux)}.$$

For calculation formula (7) above is used. Integral from (7) is calculated by numerical integration.