Intellectual Aesthetics Of Scientific Discoveries In the World of Cognitive Reality.

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Abstract. – Basing on the conceptions of Cognitive Computer Graphics (CCG) and Cognitive Reality (CR), a prototype of the multi-media CCG-System, **SVR-ANT** (the CCG-**S**ystem of **CR** for **A**dditive **N**umber **T**heory), was developed. The system allows a human being to be plunged into the color-musical world of mathematical abstractions in order to see, to look at, to touch, and to manipulate them by visual, musical, semantical, and aesthetic channels. The main aim of the system is to activate the creative intuition and the right-hemispheric, visual thinking of a human being and to help him to generate a new scientific knowledge. A lot of new and unexpected scientific CCG-discoveries was made using this CCG-Technology of VR. Some of them are described and showed in this paper.

Keywords: Scientific Visualization, Virtual and Cognitive Reality, Cognitive Computer Graphics, Mathematics, Logic, Cellular Automata, Creativity, Discovery.

1. INTRODUCTION

Some years ago, we worked out a multi-media *system* of *Cognitive Reality* based on the so-called Cognitive Computer Graphics (CCG) conception [1] which 1) is a quite non-trivial version of the well-known today Scientific Visualization, 2) is based on a *semantical* visualization of scientific abstractions (objects, their relations, etc.), 3) allows a human being to be plunged into the color-musical world of mathematical abstractions in order to see, to look at, to touch, and to manipulate them by visual, musical, semantical, aesthetic, and even ethic channels [2], etc. Such the CCG-Technology for scientific discoveries has the very practical and concrete aim: to cognize the Mathematics and Logic Foundations much more deeply, to produce registered scientific discoveries, and to create really an essentially new scientific knowledge.

Indeed, the CCG-Technology already gives a profound effect in the Basic Science.

A lot of real unique mathematical and logical discoveries were made by means of such the CCG-Technology [3-6]. Every CCG-discovery is a visual color-musical story about quite abstract mathematical ideas, hypotheses, theorems, etc., together with CCG-method itself of their real detection. Of course, then all these CCG-discoveries are proved in the rigorous mathematical sense.

In the proposed report, the latest of such CCG-discoveries is described. The question is about a generalization of some problems of Classical Mathematics and Logic, on *new foundamental propery* of the common Natural Numbers, *geometrical objects*, which *run* and *leap* along the *common*, well known series of the *common natural numbers*: 1,2,3, …

Why mathematicians could not ever see these objects during about 3000 years? - Because, in the *common* series 1,2,3, . . ., these new geometrical objects (parabolic solitons, Fibonacci's triangles, etc.) are *virtual* objects. And they become really *actual*, *visible*, and *accessible* to rigorous mathematical investigations in the *Virtual Reality* CCG-Space only.

The most wonderful is that some of these virtual objects were detected by not a professional mathematician, but by a professional…artist and a professional…chemist! - Of course, by means of just the CCG-Technology! (more info: http://www.com2com.ru/alexzen/vgeom/vgeom.html)

The Natural Numbers series 1,2,3, … , is a basic object not only of Mathematics. It is an important element of all millennials Humankind Culture as a whole. Therefore, we believe that the CCG-Technology for the *purposeful* scientific creativity amplification and the CCG-discoveries already obtained in the area of the common Natural Numbers will have a *profound effect* not only in Mathematics, but also in Philosophy and Psychology of scientific cognition, in knowledge management, and, especially, in scientific *education via discoveries*. The CCG-Technology is also an effective manner to discover a new, high effective, non-standard *strategies* for problem solving and decision making in industrial, economical, financial, political, etc. areas [7-9].

At last, all that is simply beautiful from the scientific, educational, and artistic point of views.

2. DEFINITION OF THE PYTHOGRAM

Df.1. The Pythogram is a Color-Musical 2D-Image of an Abstract Number-Theoretical (NT) Object.

Df.2. NT-Object is a segment of the 1D-Series of Natural Numbers, $n \geq 1$, with a NT-Predicate $P(n)$ defined on it.

Df. 3. The Sense of the Number Theory consists in (by B.N.Delone) the very difficult to comprehend Connection between the Additive and Multiplicative Properties of Natural Numbers.

2.1. CCG-Technique.

- Color all natural numbers in according with the rule: if $P(n)$ then White else Black.
- Convert the 1D-series of natural numbers into the 2D-image (table).
- Make Musical such 2D-image in according with a function: $F(P(n)$, place, value, any other NT-Properties of n, and so on).

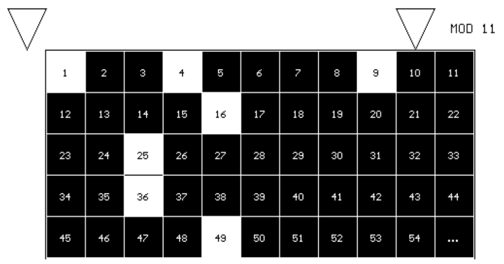


Fig.1. Pythogram of a segment [1,54] of the Natural Number series by modulus 11. Here, NT-Predicate $P(n)$ = "n *is a square of a natural number*".

2.2. Main Pythogram Properties:

- *Additivity* of n is modeled by its *colour*.
- *Multiplicativity* of n is modeled by its *position* in a j-coloumn of the Pythogram since for any fixed mod L, [if n ∈ j-column, then n ≡ j (mod L)].
- So, any pythogram as a whole *visualizes* the unique *twice abstract* Connection between two *abstract* properties of the Natural Numbers - there *additivity* and *multiplicativity* properties.
- Changing modulus we get a unique possibility *TO SEE* this Connection (the *SENSE* of the Number Theory, by B.N.Delone) *in Dynamics*.
- Any pythogram is a Musical Invariant of an abstract mathematical structure.

2.3. Cognitive Explanation Of The CCG-Visualization Technology

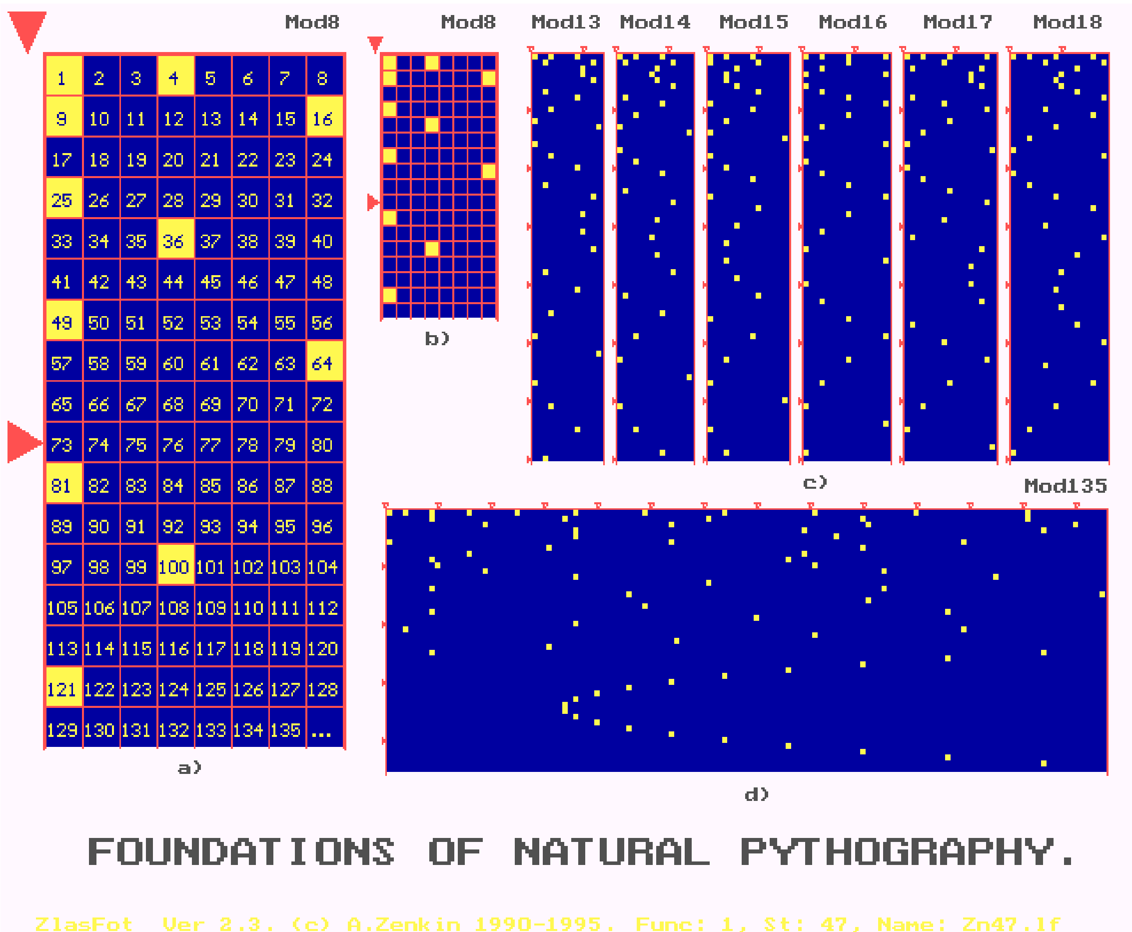


Fig. 2. The pythogram of the number-theoretical predicate,
P(n) = “n *is a square of a natural number*”.

1. Consider the simplest number-theoretical predicate: P(n) = "n is a square of a natural number".
2. Fix any modulus, say, mod=8, and visualize the predicate P(n) in the segment [1,135]. We produce the CCG-pythogram of P(n) (Fig.2a).
3. Decreasing the size of cells, we enlarge the field of vision (Fig.2b-d).
4. Changing the modulus of the pythogram, we produce a color-musical movie which allows us to see a new and unexpected dynamical properties of abstract mathematical structures. Here – of the predicate P(n).

3. INTELLECTUAL CCG-INSIGHTS INTO THE COGNITIVE WORLD OF NATURAL NUMBERS

3.1. Beauty And Truth Of Mathematical Abstractions.

In the far "non-computer" epoch, great Gotfried Leibniz supposed that "figures are useful *to awake a thought*". The modern computer technologies open unique opportunities just for awakening a non-traditional mathematical thought.

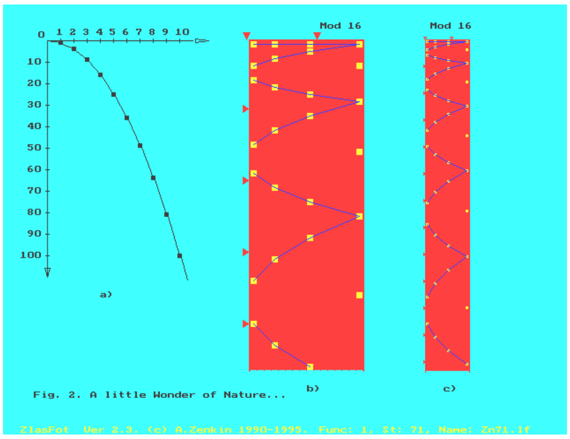


Fig. 3. The most beautiful and unexpected CCG-Discovery: traditional (a) and non-traditional (b,c) forms of the visual representation of the same mathematical object - of the well-known natural squares set, {1, 4, 9, 16, 25, 36, ...}. We have got a certain new "paradox" in modern mathematics: distance between a) and b) is equal to ... about 2000 years! To draw b) could even Pythagoras. In 1841, Meobius drew very similar parabolas, even by the modulus 16 (!), in his known nomographical works. But only CCG allows us to see in the first time this fantastic transformation having a deep cognitive psychological and philosophical sense !

3.2. The Great Pythagoras' Dream: The World As A Harmonic Unity Of Number, Image, And Music.

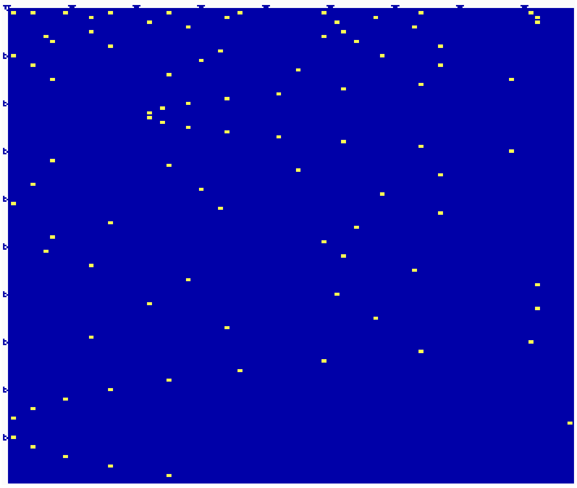


Fig. 4. A frame of the CCG-movie about a lot of unknown dynamic properties of the well-known series (1) of natural numbers squares. - If you are not in raptures about seeing this highest intellectual-aesthetic Wonder of the Natural Numbers World (almost by H.Hesse's "Das Glassperlenspiel": remember about NT-Music!), then Mathematics is not your vocation...

During more than 3000 years, the Humankind is "staring at" the such trivial and exhaustively investigated mathematical object, as the set of squares of natural numbers: **1**,2,3,**4**,5,6,7,8,**9**,10,11,12,13,14,15,**16**,17, … The pythogram modulus is a unique degree of our freedom: changing the modulus, we create a CCG-movie, which is not only interesting from the point of view of the mathematical and intelligent aesthetics, but which permits us *to see* new *dynamic properties* of abstract NT-structures. As a rule, such the dynamic mathematical properties are simply incomprehensible in statics! One of the frames of this CCG-movie (by mod=87) is shown in Fig.4. We can *see* there a number of surprising new and non-trivial NT-facts (virtual geometrical objects).

3. CCG-GENERALIZATION Of CLASSICALWARING'S PROBLEM (1770 - 1980)

In the far 1770, the English mathematician Edward Waring formulated his famous hypothesis on representation of natural numbers n≥1 as the sums of the kind: $n = \sum_{i=1}^{s} n_i^{\,r}$, where r≥2, s≥1 - are integer, and all n_i are *non-negative* integers: 0,1,2,3,...

Table 1. Logical and semantical isomorphism of classical and non-classical Waring's problem.

CLASSICAL WARING's PROBLEM (D.Hilbert, 1909)	GENERALIZED WARING's PROBLEM (A.Zenkin, 1979)
For the fixed m = 0 and for every r ≥ 2 there exists: 1) the smallest number of summands, g(r) ≡ g(0,r), - such that for any s ≥ g(0,r): N(0,r,s) = ∅	For any m = 1, 2, 3, ... and for every r ≥ 2 there exist: 1) the smallest number of summands, g(m,r), 2) the finite invariant set, Z(m,r) ≠ ∅, such that for any s ≥ g(m,r): $N(m,r,s)=\{s\cdot m^r+z:z\in Z(m,r)\}$

Euler, Lagrange, Gauss, Legendre and many other outstanding mathematicians of XVIII-XX cc. (which were possessed of the eminent scientific intuition!) were investigating the Classical Waring Problem (CWP) during more than a hundred years. However, only in the 1909, the greatest German mathematician David Hilbert gave the complete solution of CWP (see Table 1, the left column).

In general, the traditional mathematics " was staring " at CWP more than 200 years. However, only CCG has allowed us *to see* the completely unexpected *fact*, that the CWP represents only the 0-floor of a much more general ∞-floor problem - so called Generalized Waring's Problem (GWP) (see Table 1, the right column). - Why not "a relativity theory" of the traditional mathematical values! - Can't we now formulate (and prove!) the famous classical (m=0) Lagrange theorem for every m=1,2,3,...?!

3.2. From Classical To Generalized Waring's Problem.

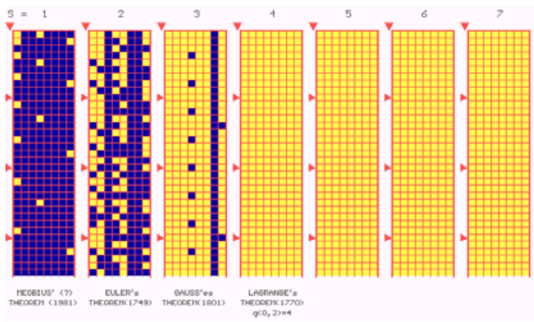


Fig. 5. Classical sums of squares.

GAUSS' THEOREM (1801), s=3.
$\forall$ n $\geq$ 1 { *if* n = $(8k+7)\cdot 4^l$, k, l = 0, 1, 2, ...
then n $\in$ N(0,2,3) }, *i.e.* |N(0,2,3)| = ∞.

LAGRANGE'S THEOREM (1770), s≥4. Any n is a sum of four squares, *i.e.*, g(0,2) = 4 and N(0,2,4) = $\varnothing$.

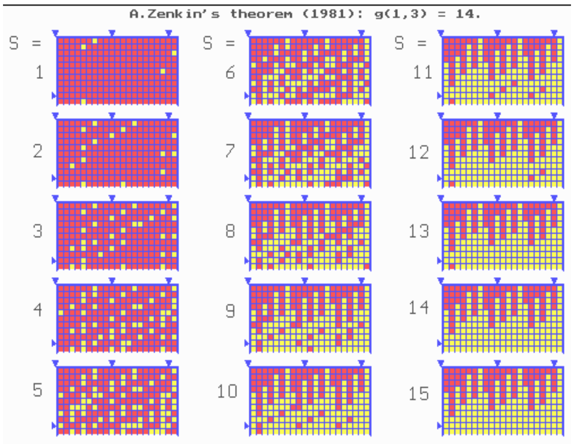


Fig. 6. Non-Classical sums of cubes.

A.ZENKIN's THEOREM (1979). For any s $\geq$ 14, any natural number n $\geq$ 1 is representable as a sum of exactly s cubes of positive integers, except for the numbers 1,2,3,..., s-1, and the numbers of the form s + Z(1,3), where the set, Z(1,3), is known and can be seen here directly and explicitly.

4. SUPERINDUCTION: NEW LOGICAL METHOD FOR MATHEMATICAL PROOFS WITH A COMPUTER

In **1949**, German mathematician H.E.Richert proved the following quite strange inductive statement: "IF there exists a natural number, say, n* such that Q(n*) is true THEN for any natural number n>n* P(n) is true", or in a short symbolic notation:

$$[\exists n^* Q(n^*)] \rightarrow [\forall n{>}n^* P(n)], \quad (1)$$

where P and Q=f(P) are two collections of number-theoretical properties of common finite natural numbers (or predicates given on the natural numbers set).

So, the H.E.Richert Theorem (further - EA-Theorem) is a mathematical, i.e., authentic, proof of the inductive statement of the quite unusual form: "from a *single* statement, [∃n*Q(n*)], to a *common* one, [∀n>n*P(n)]".

In **1978**, using Cognitive Computer Visualization of mathematical abstractions technology, we discovered two new different *classes* of such the EA-Theorems and formulated the **Super-Induction** (SI) method [1]. By means of the SI-method, a lot of conceptually new scientific results was obtained in Classical Number Theory [1-4]

Note some unexpected connections of SI-method with some basic logical conceptions.

1. According to ***inductive*** J.S.Mill's Logic, we always can formulate a *common* statement, say, H basing on a set of *particular* facts, but such H will always be only a *plausible* statement. The existence itself of EA-Theorems (1) and SI-method show that the main inductive Logic paradigm is broken in some areas of discrete mathematics.

2. The "***MODUS PONENS***" **RULE** sounds so: [A&[A→B]] → B. Mathematical Logic and meta-Mathematics consider the implication [A→B] as a *deductive* inference of a *less common* consequence B (e.g., a theorem) from a *more common* premise A (e.g., an axiom system). SI-method generalizes the "modus ponens" rule to the case when the premis A is a *single* statement, but the consequence B is a *common* one.

3. It can be easy shown, that the SI-method generalizes the classical complete mathematical induction B.Pascal's method. Moreover, SI-method works well there where B.Pascal's method simply does not work.

4. Cognitive Visualization of mathematical abstractions and SI-method allow, by certain conditions, to use corresponding *cognitive images* as quite legitimate arguments in rigorous mathematical proofs, i.e., they realize an ***authentic ostensive* proofs** in, say, L.E.J.Brouwer's sense [1-4].

4.1. The Finiteness Criterion For The Invariant Sets, Z(m,r). Example 1.

THEOREM 1. For any m $\geq$ 1 and r $\geq$ 2,

IF **∃n*Q(n*)**

where Q = a) & b) & c) and
a) n* $\in$ Z(m,r);
b) n* + i $\notin$ Z(m,r) for any i = 1, 2, ... , k;
c) k $\geq$ $(m + 1)^r$ - m^r ,

THEN **∀n>n* P(n)**, where P(n) = " n $\notin$ Z(m,r) ".

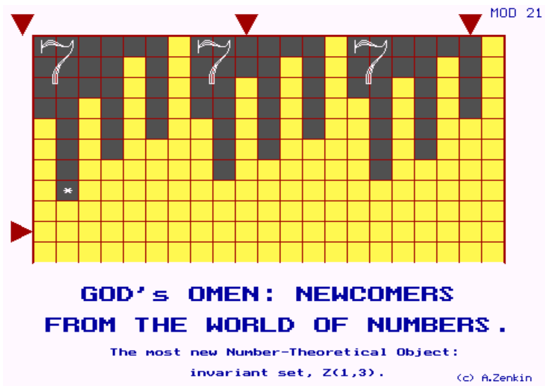


Fig. 7. Pythogram of the invariant set, Z(1,3) in GWP.

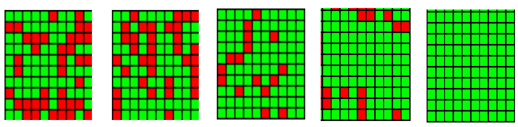
By means of the most power Habble's Telescope, modern Sciencesearches for New-Comers in the heart of the far Cosmos…

But our high-intelligent CCG-"Telescope" has found them in the very beginning of the common series of the Godlike Natural Numbers... Just these wise New-Comers pointed the way to the beautiful Cognitive Reality World of Natural Numbers and helped us to make a lot of wonderful CCG-Discoveries.

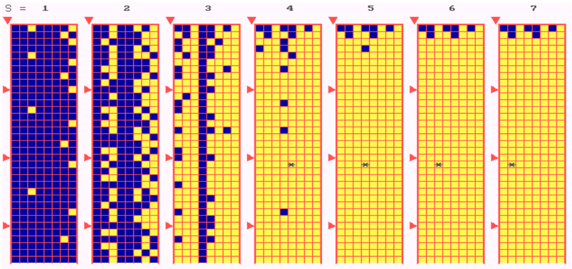
5. CLASSICAL NUMBER THEORY AND CELLULAR CCG-AUTOMATA

A NEW CCG-VERSION OF JOHN CONWAY'S

"THE GAME OF LIFE"



A sequence of the "Life"-configurations



Visual Proof of Pall's theorem: g(1,2) = 6 by module 8.
Here: *-symbol in the 21-th row marks the outstanding number n*(1,2)=169,
and the first number in the every s-th pythogram is s+1, s=1-7.
(c) A.Zenkin

G.PALL'S THEOREM 1 (1933).

. $g(1,2) = 6$, *and* $|N(1,2,5)| < \infty$.

DESCARTES' THEOREM (XVIII C.).

$\forall\, n \geq 1$ [*if* $n = (2, 6, 14)\cdot 4^l$, $k, l = 0, 1, 2, \ldots$ *then* $n \in N(1,2,4)$ }, *i.e.* $|N(1,2,4)| = \infty$].

5.1. Waring's Problem As A Cellular Automaton.

All additive Number Theory can be re-formulated in the cellular automaton language in the following way.

In general, our cellular automaton field is a two-dimensional matrix (table) with M column (the modulus of the CCG-image) and the **infinite** numbers of strings.

In practice (due to natural computer graphics limits) the modulus M and a number of strings N are limited by not very large finite values. However, using a window-technique, we can visualize and look through quite large and distant segments $[n_1, n_2]$ of the Natural Numbers series, or what is the same, - of that cellular automaton field.

CONFIGURATION. For any m=0, r≥2, s=1,2,3, … , P(n;r;s) = "n is a sum of s r-th powers",
TRANSFORMATION RULE. **IF** P(n; r; s) is true **THEN** $P(n+k^2; r; s+1)$ is true too, where k=0,1,2, …

(for more info: http://www.com2com.ru/alexzen/WEB-2000/CCGandCA/CCGandCellularAutomata.html)

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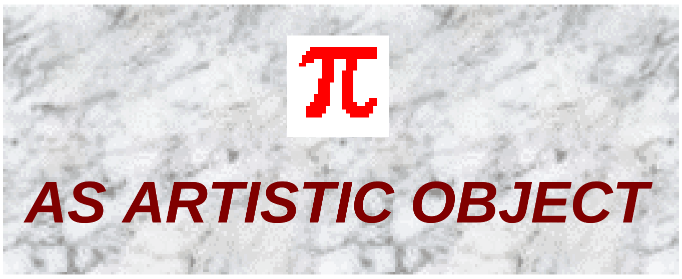
APPENDIX.



INTELLECTUAL AESTHETICS OF MATHEMATICAL ABSTRACTIONS

CCG-TECHNOLOGY for COGNITIVE SEMANTIC SCIENTIFIC VISUALIZATION

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123456789*123456789*123456789*123456789*123456789*
31415926535897932384626433832795028841971693993751
05820974944592307816406286208998628034825342117067
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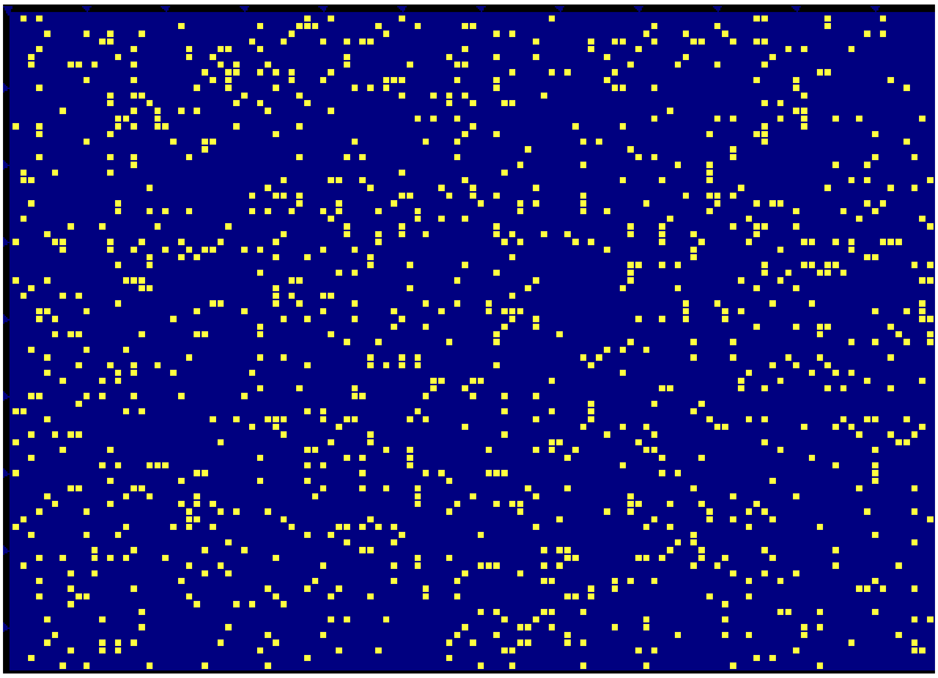

Pythogram of the π-Number's **1**-Digit.
<It's being read (counted) from left to right, and from top to bottom>

> **"IF YOU SCRUTINIZE INTO THE ABYSS NARROWLY AND FOR A LONG TIME, THEN THE ABYSS ITSELF BEGINS TO PEER AT YOU . . ."**

(c) Alexander Zenkin, e-mail: alexzen@com2com.ru
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"*Infinitum Actu Non Datur*" - **Aristotle**.
"*Drawing is a very useful tool against the uncertainty of words*" - **Leibniz**.