Nonorthogonal RGB-space decomposition

Natalia Vaganova
Institute of Computational Mathematics and Mathematical Geophysics,
Novosibirsk, Russia
vaganova@oapmg.sscc.ru

Abstract

This paper deals with some new ideas of image color models constructions. The results of spherical image coding with the division are also presented.

***Keywords: Image compression**, Color Space.*

1. INTRODUCTION

At present time the more or less image data interpretation procedure presents significant interest for the specialists in image processing and data transmission by communication channel. It is clear the main question to decide what kind of information we have to extract from huge volume data in order to catch sense theirs.

The more common way for solution of this problem based on separation of typical image properties. This process corresponds to brain processing, that is all of unnecessary or obscure information is discarded and the sketch is made up. This sketch then is used as image synthesizing, contained only main properties of concrete image.

2. 3FA IN STATISTICS OF USE COLOR IN THE CODING IMAGE

Let us consider the image palette

$$\pi_N = \{P_i = (R_i, G_i, B_i), i = 1, 2, ..., N\}$$

in RGB color space and . R_i, G_i, B_i are non-negative integers. We will determine a gravity centre $P^0(R^0, G^0, B^0)$ of the palette by next coordinates:

$$R^0 = N^{-1}\sum_{i=1}^{N} R_i, \quad G^0 = N^{-1}\sum_{i=1}^{N} G_i, \quad B^0 = N^{-1}\sum_{i=1}^{N} B_i .$$

The main direction ν in palette π_N by definition is the directing vector of straight line

$$L = \{P \in R^3 : P = P_0 + t\nu, -\infty < t < +\infty\}$$

contained the gravity center and has minimal root-mean-square distance for points of the palette. We will suppose vector ν has unit Euclidian length - $\|\nu\|_2^2 = 1$. Hence we have to minimize the functional $\Phi(\nu) = \sum_{i=1}^{N} \rho(L, P_i) \to \min_{\nu}$; here $\rho(L, P_i)$ means the Euclidian distance between the line L and palette point P_i. After the simplest linear transformation $P_i' = P_i - P^0$ we have $\rho^2(L, P_i) = \rho^2(L', P_i') = \|P_i'\|^2 - (P_i', \nu)^2$. Then we get the minimization problem

$$\Phi(\nu) = \sum_{i=1}^{N}\left[\|P_i'\|^2 - (P_i', \nu)^2\right] \to \min_{\nu}$$

under the natural constrain $\|\nu\|_2^2 = 1$, or the maximization problem arises $\psi(\nu) = \sum_{i=1}^{N} (P_i', \nu)^2 \to \max_{\nu}$. Using the Lagrange function $L(\nu, \lambda) = \psi(\nu) + \lambda(1 - \|\nu\|^2)$ after the differentiation we obtain

$$\frac{1}{2}\frac{\partial L}{\partial \nu_k} = (\sum_{i=1}^{N} x_i^{(k)} P_i', \nu) - \lambda \nu_k = 0, \; k = 1, 2, 3,$$

where $x_i^{(1)} = R_i - R^0$, $x_i^{(2)} = G_i - G^0$, $x_i^{(3)} = B_i - B^0$, $i = 1, 2, ...N$. If we denote by B the rectangular $N \times 3$-matrix $B = \left\{x_i^{(k)}\right\}_{i=\overline{1,N}, k=\overline{1,3}}$, then relation above can be written in the matrix form

$$B^T B \nu = \lambda \nu \qquad (*),$$

i.e., ν is the eigenvector of $B^T B$ and $\|\nu\|_2^2 = 1$. It is also worth to mention that the $\nu^{(1)}$, $\nu^{(2)}$, $\nu^{(3)}$ eigenvectors produce an orthonormal basis because the matrix $B^T B$. Symmetric. Let $B^T B \nu^{(l)} = \lambda_l \nu^{(l)}$, $\lambda_1 \ge \lambda_2 \ge \lambda_3 \ge 0$, $l = 1, 2, 3$.

Let us find $\psi(\nu^{(l)})$,

$$\psi(\nu^{(l)}) = \sum_{i=1}^{N}\left(\sum_{k=1}^{3} x_i^{(k)}, \nu_k^{(l)}\right)^2 = (B^T B \nu^{(l)}, \nu^{(l)}),$$

$$\psi(\nu^{(l)}) = \left\|B \nu^{(l)}\right\|^2 = \lambda_l^2 .$$

Thus, the maximal value of $\psi(\nu)$ or minimal value of $\Phi(\nu)$ take place on the eigenvectors $\nu^{(1)}$ which corresponds to the maximal eigenvalue λ_1 of the $B^T B$ matrix.

If we construct now the plane which includes gravity center P^0 of the palette π_N and is orthogonal to the eigenvector $\nu^{(1)}$, and find in this plane the best direction, then it corresponds to the second eigenvector $\nu^{(2)}$ for the matrix $B^T B$. The last eigenvector $\nu^{(3)}$ determines the least important direction in the palette π_N. Thus, the eigenvectors $\nu^{(1)}$, $\nu^{(2)}$, $\nu^{(3)}$ determine three factors in the order of "important decay" with respect to the a priory given palette π_N and the levels of "importance", as we suppose, are regulated by the ratio $\lambda_1 : \lambda_2 : \lambda_3$.

According to this 3-factors analysis of the palette the color RGB-image can be decomposed into 3 components (3FA-decomposition) F_1, F_2, F_3 by the following rule:

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} \nu_1^{(1)} & \nu_2^{(1)} & \nu_3^{(1)} \\ \nu_1^{(2)} & \nu_2^{(2)} & \nu_3^{(2)} \\ \nu_1^{(3)} & \nu_2^{(3)} & \nu_3^{(3)} \end{pmatrix} \begin{pmatrix} R - R^0 \\ G - G^0 \\ B - B^0 \end{pmatrix},$$

or the matrix form $F = V(P - P^0)$. The inverse transformation is also very simple $P = P^0 + V^T F$, because V is an orthogonal matrix. This kind of decomposition for the color image we call 3FA standard.

3. 3FA-DECOMPOSITION BASED ON IMAGE PREPROCESSING AND INTERACTIVE USER ADDITIONAL INFORMATION

The algorithm, described in previous section may be really used on practice and can be applied for effective decomposition of images of average resolutions. Also, there are numbers of images in that the color are not the main factors from the point of the palette, however one may be important for image understanding. For example it may be some dedicated image details, captions, geographic object on the map and so on.

In this case for effective image analysis and decomposition the additional information from image assigned with details has been given. If there many important details are on the image the information may be taken from image analysis by histogram construction and we can say the color of given number are very often meeting in the image and it may be important for decoding of the image. User also may want importance coefficients assign to colors in the palette. That is needs to propose algorithms, where image colors include in 3FA-analysis with some weight.

The method of image decomposition with additional information is similarly to above described method. In contrast to problem (*) the color statistics correspond to weight coefficients.

Let us consider next problem: in color RGB space with given finite palette π_N we take initial image as number of point $\{P_i = (\alpha_i(R_i, G_i, B_i)),\ i = 1, 2, \ldots N\}$, here R_i, G_i, B_i - intensities of Red, Green, Blue color components corresponds to point of the palette and α_i - weight i.e. number of points of i-th color in the image.

Therefore the Gravity Center of π_N may be easy calculated:

$$R^0 = N^{-1} \sum_{i=1}^{N} \alpha_i R_i,\ G^0 = N^{-1} \sum_{i=1}^{N} \alpha_i G_i,.$$

$$B^0 = N^{-1} \sum_{i=1}^{N} \alpha_i B_i$$ and (*) may be rewrite

$$B^T \cdot AL \cdot B\nu = \lambda \nu,$$

where B is rectangular $N \times 3$ matrix from previous section, AL - vector at N length consist of weight coefficients. All described in previous section reasoning are similarly performed.

Remark.

If reader not assigns colors from palette special weight, that is computer performs calculations automatically, then palette from the image are not picked out since it decreases times complexity of the method as for image with average resolution as for high one.

4. NONORTHOGONAL DECOMPOSITION OF RGB SPACE

The ways described above find three orthogonal vectors. In real time as we understand always the vectors has not been orthogonal. Of course, the principal direction in this case is found in a better way and corresponds to maximal energy concentration in the first component, but second and third vectors may be selected in the best of manner and time complexion is naturally increased.

Well let us principle direction have been selected and first component performed

$$F_1 = (\nu_1^{(1)}, \nu_2^{(1)}, \nu_3^{(1)}) \cdot \begin{pmatrix} R - R^0 \\ G - G^0 \\ B - B^0 \end{pmatrix}.$$

Then inverse transformation gives the components of the images reconstructed by factor F_1:

$$R_{F_1} = R^0 + \nu_1^{(1)} F_1,$$
$$G_{F_1} = G^0 + \nu_2^{(1)} G_1,$$
$$B_{F_1} = B^0 + \nu_3^{(1)} B_1.$$

Calculates the differences between initial image components R_I, G_I, B_I and reconstructed by factors F_1 to get the second direction $\tilde{R} = R_I - R_{F_1}, \tilde{G} = G_I - G_{F_1}, \tilde{B} = B_I - B_{F_1}$. Further, considering $\tilde{R}, \tilde{G}, \tilde{B}$ instead of initial image components R_I, G_I, B_I selects the principal direction

$$F_2 = (\nu_1^{(2)}, \nu_2^{(2)}, \nu_3^{(2)}) \cdot \begin{pmatrix} \tilde{R} - \tilde{R}^0 \\ \tilde{G} - \tilde{G}^0 \\ \tilde{B} - \tilde{B}^0 \end{pmatrix}.$$

Inverse transformation performs the image components reconstructed by factor F_2

$$R_{F_2} = \tilde{R}^0 + \nu_1^{(2)} F_2,$$
$$G_{F_2} = \tilde{G}^0 + \nu_2^{(2)} G_2,$$
$$B_{F_2} = \tilde{B}^0 + \nu_3^{(2)} B_2.$$

At last third direction is selected by analogue with second one only for $\tilde{\tilde{R}} = \tilde{R} - R_{F_2}, \tilde{\tilde{G}} = \tilde{G} - G_{F_2}, \tilde{\tilde{B}} = \tilde{B} - B_{F_2}$ initial image. Original image is restored by formulas:

$$R_I = R_{F_1} + R_{F_2} + R_{F_3},$$
$$G_I = G_{F_1} + G_{F_2} + G_{F_3},$$
$$B_I = B_{F_1} + B_{F_2} + B_{F_3}.$$

5. COMPARISON OF PAL AND 3FA STANDARDS WITH RESPECT TO FRACTAL IMAGE COMPRESSION BASED ON SPHERICAL OBJECT CLASSIFICATION.

PAL is a famous television oriented standard [2,3,4]based on transformation

$$\begin{pmatrix} Y \\ U \\ V \end{pmatrix} = \begin{pmatrix} 0.30 & 0.59 & 0.11 \\ -0.15 & -0.29 & 0.44 \\ 0.62 & -0.51 & -0.10 \end{pmatrix} \cdot \begin{pmatrix} R \\ G \\ B \end{pmatrix},$$

which tends to forthcoming compression. As the compression algorithm we use here so called fractal image compression based on spherical classification. The algorithm consists of special classification of greylevel square regions with B-splines approximation as postprocessing of the image and is detailed described in [1]. We execute the comparison of PAL and 3FA standards in this context with three kinds of images: "Lena", usual life color photos "Cheetah" and frame from television broadcastings video [5].

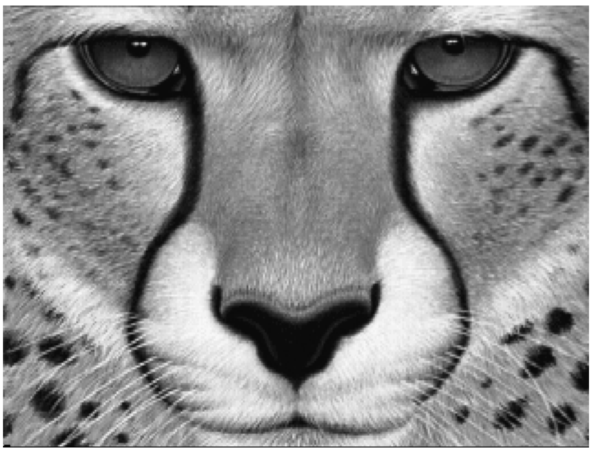

Figure 1. First decoded component of CHEETAH.

Unfortunately, it is impossible here to print color pictures, and we described half-tone components only. The image in 3FA-decomposition and in PAL-decomposition will be compressed with the same compression ratio.

Example 1. CHEETAH.

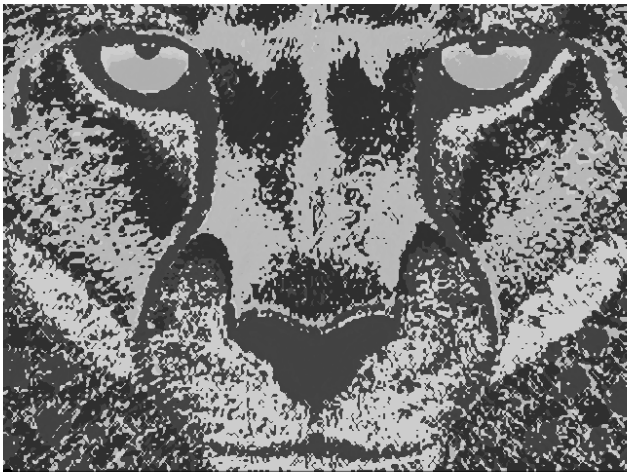

Figure 2. Second decoded component of CHEETAH.

Initial image has resolution 640×480 with 8-bit color depth.

Figure 3. Third decoded component of CHEETAH.

Three factor analysis of the image gives the following results:

$$P^0 = (149.23, 114.56, 100.96),$$
$$\lambda_1 \div \lambda_2 \div \lambda_3 = 2312 \div 266 \div 10,$$
$$V = \begin{pmatrix} 0.56 & 0.62 & 0.55 \\ -0.72 & -0.05 & 0.69 \\ 0.40 & -0.79 & -0.47 \end{pmatrix}.$$

Example 2. LENA.

Initial image has resolution 512×512 with 16-bit color depth. Three factor analysis of the image gives the following results:

Figure 4. First decoded component of LENA.

Figure 5. Second decoded component of LENA.

$$P^0 = (181.89, 104.32, 110.79),$$
$$\lambda_1 \div \lambda_2 \div \lambda_3 = 8324 \div 908 \div 106,$$

$$V = \begin{pmatrix} 0.59 & 0.70 & 0.40 \\ -0.71 & 0.21 & 0.67 \\ 0.38 & -0.68 & -0.62 \end{pmatrix}.$$

Example 3. VIDEOFRAME.

Initial image has resolution 384×288 with 24-bit color depth.

Figure 6. Second decoded component of LENA.

Three factor analysis of the image gives the following results:

Figure 7. First decoded component of VideoFrame.

$$P^0 = (150.64, 115.38, 111.07),$$
$$\lambda_1 \div \lambda_2 \div \lambda_3 = 829 \div 747 \div 419,$$
$$V = \begin{pmatrix} 0.58 & 0.58 & 0.57 \\ -0.63 & -0.11 & 0.77 \\ 0.50 & -0.81 & -0.30 \end{pmatrix}$$

Figure 8. Second decoded component of VideoFrame.

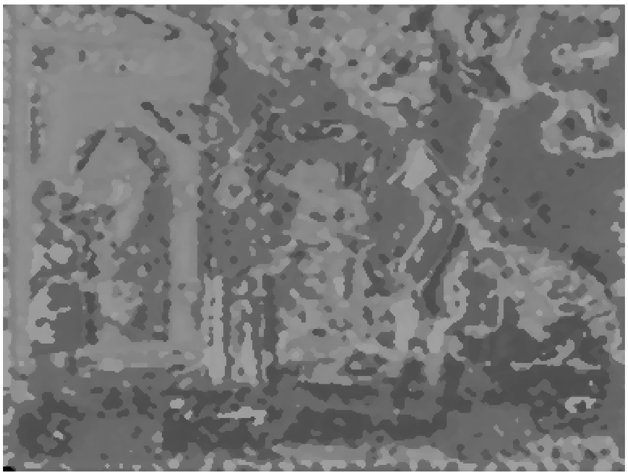

Figure 9. Third decoded component of VideoFrame.

The carry out experiments have shown that the application of 3FA decomposition increases the image quality (PSNR) of decoding images by 2-4 dB with respect to PAL standard. The results of 3FA non-orthogonal decomposition are presented at figure 1-9. The experimental data about orthogonal using may be found in [5].

6. CONCLUSION

In our experiments with decomposition and compression of the color images we have not the aim to get the exact answer to the question: what is the better, the well-known PAL-standard or proposed 3FA-method? In fact, our experiment has shown that 3FA is usually better, and it is not surprising because the additional information on the palette was used in decomposition. On the other hand PAL-standard is more universal and has the same coefficients for all pictures and palettes.

It is dangerous to compress the main components F_1 and Y both in 3FA and PAL but rest components can be essentially compressed without loss of quality. The components F_1 and Y are quite informative in the color image, and the rest components which provide the beautiful colouring of the word around us, are not essentially important.

The experimental compression ratio results with respect to JPEG2000 (the compression ratios are about 1.5-2 times better then JPEG2000 in the same PSNR of decoded images) algorithm are comprehensively described in [1].

The author is supported by Russian Science Support Foundation, by integration project №6 “The development approximation theory by splines, trigonometric polynomials and fractals with application to construction of water turbine models and broadcasting image compression”, by Interschool Scientific-Technical Program “Fundamental higher-school research in the field of natural sciences and humanities. Universities of Russia” under Grand №03.01.195.

7. REFERENCES

[1] Vaganova N.A. The algorithm of Variance and Spherical fractal base construction for image compression. Vestnik of Novosibirsk State University, series: mathematics, mechanics and informatics, 1(2001), NSU, page 23-39. [In Russian].

[2] Gruzman I.S., Kirichuk V.S., Kosyh V.P. Peretyagin G.I., Spector A.A. Digital image proceedin in the information system, NSU, 2000.

[3] Zubarev Yu.G., Gloriozova G.L. The broadcasting technique. Guide, Moskow:-Radio and communication, 1994.

[4] Brodsky M.A. The Color broadcasting, second edition, Moskow:-High school,1994

[5] Vaganova N.A. The fractal video compression with spherical classification and 3FA-orthogonal preprocessing. Vestnik of Novosibirsk State University, series: mathematics, mechanics and informatics, 11page, 2005. [In Russian, in print].

About the author

Natalia Vaganova is a scientific researcher at Institute of Computational Mathematics and Mathematical Geophysics, Department of Numerical analysis and Computer Graphics, Head researcher of fractal group, Ener1Group Inc. (USA)

Natalia Vaganova has received her Ph.D. in December, 2002. Her contact emails are vaganova@oapmg.sscc.ru, nvaganova@enersoft.com.