Feline: Fast Elliptical Lines for Anisotropic Texture Mapping

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Abstract

Texture mapping using trilinearly filtered mip-mapped data is efficient and looks much better than point-sampled or bilinearly filtered data. But trilinear filtering represents the projection of a pixel filter footprint from screen space into texture space as a square, when in reality the footprint may be long and narrow. Consequently, trilinear filtering severely blurs images on surfaces angled obliquely away from the viewer.

This paper describes a new texture filtering technique called Feline (for Fast Elliptical Lines). Like other recent hardware anisotropic filtering algorithms, Feline uses an underlying space-invariant (isotropic) filter with mip-mapped data, and so can be built on top of an existing trilinear filtering engine. To texture a pixel, it uses this space-invariant filter at several points along a line in texture space, and combines the results. With a modest increase in implementation complexity over earlier techniques, Feline more accurately matches the desired projection of the pixel filter in texture space, resulting in images with fewer aliasing artifacts. Feline's visual quality compares well against Elliptical Weighted Average, the best software anisotropic texture filtering algorithm known to date, but Feline requires much less setup computation and far fewer cycles for texel fetches. Finally, since it uses standard mip-maps, Feline requires minimal extensions to standard 3D interfaces like OpenGL.

CR Categories and Subject Descriptors: I.3.1 [Computer Graphics]: Hardware Architecture – Graphics processors; I.3.7 [Computer Graphics]: Three-dimensional Graphics and Realism – Color, shading, shadowing, and texture

Additional Keywords: texture mapping, anisotropic filtering, space-variant filtering

1 INTRODUCTION

Ideally, computing a textured value for a pixel involves perspective projecting a filter from screen space (indexed by *x* and *y* coordinates) into texture space (indexed by *u* and *v* coordinates), then combining this with a reconstruction filter to create a unified filter in texture space. Each texel inside the unified filter's footprint is weighted according to the unified filter's corresponding value in screen space, the weighted samples are accumulated, and the sum is divided by the filter's volume in texture space. Figure 1, inspired by Lansdale [8], gives an intuitive view of this process. A pixel filter is a "window" onto a portion of the texture map; the window's opacity at each point corresponds to the filter's weight. The grid represents a texture map; the shaded rectangle the screen. We view an elliptical portion of the texture map through a round pixel filter. (In degenerate cases, a circle projects to an arbitrary conic section, but for our purposes an ellipse suffices.)

Figure 2 shows a typical pixel filter in screen space—a Gaussian with weighting $e^{-\alpha (x^2 + y^2)}$, truncated to zero beyond a radius of one pixel, and with an α of 2. Tick marks on the *x* and *y* axes are at one pixel intervals; the *x*-*y* grid is at 1/10 pixel intervals. Figure 3 shows an exemplary perspective projection of this filter into texture space, where the tick marks on the *u* and *v* axes are spaced at one texel intervals, and the grid is at ½ texel intervals. We normalize all texture filter volumes to one to allow direct comparisons between graphs, then highly exaggerate the vertical axis. Note the distorted filter profile: each contour line is an ellipse, but the ellipses representing lower sample weights are increasingly offset from the filter center.

Mapping the texel positions in Figure 3 back into pixel positions in Figure 2 (let alone creating a unified filter), so that relative weights can then be applied to the texel values, is a gruesome affair. Rather than using a perspective projection, Heckbert and Greene [4][6] suggest using a locally parallel (affine) projection,

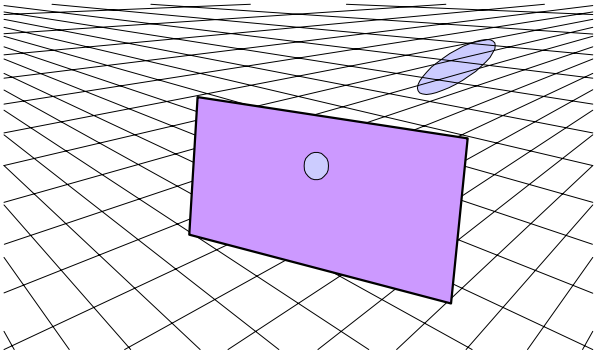

Figure 1: Viewing an elliptical texture area through a circular pixel window.

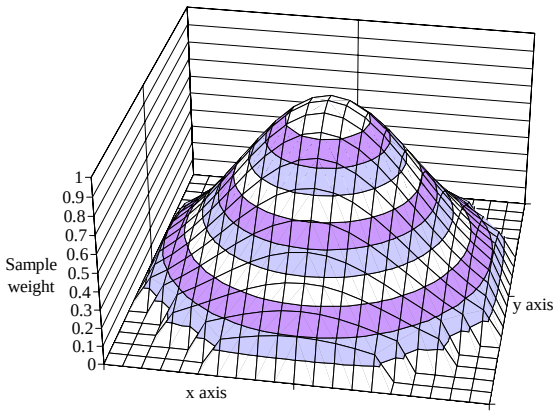

Figure 2: A circular Gaussian filter in screen space.

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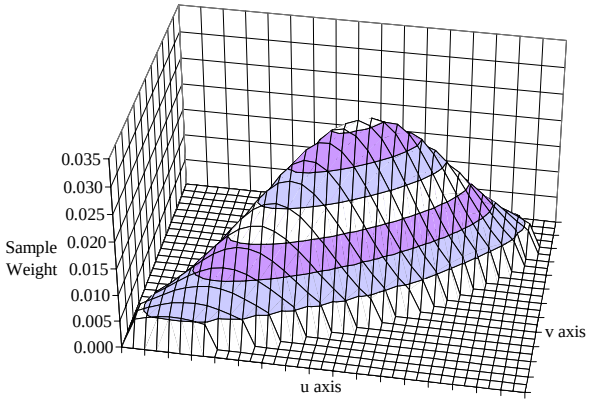

Figure 3: A perspective projection of a Gaussian filter into texture space.

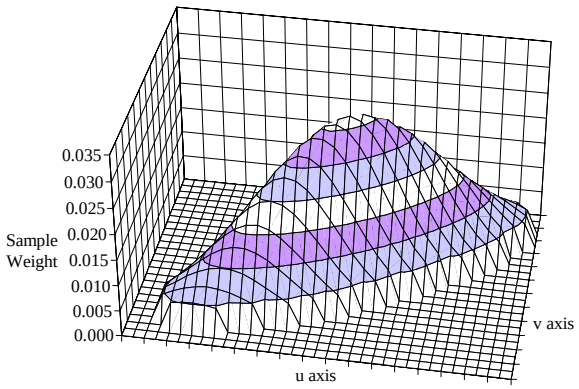

Figure 4: An affine projection of a Gaussian filter into texture space.

as shown in Figure 4. This drastically simplifies computing the footprint and weights of the projected filter. This simplification is visually insignificant. The modest weight differences between Figure 3 and Figure 4 are not detectable in images, and to get the distortion shown in Figure 3 requires a nearly edge-on view of the surface being texture mapped, in which all detail is lost anyway.

Our algorithm approximates the elliptical filter shown in Figure 4 by performing several isotropic (e.g. trilinear, Gaussian) filtering operations, called *probes*, along the major axis of the ellipse. In comparison to other hardware anisotropic filtering methods, Feline better approximates the elliptical filter by more accurately determining the length of the line along which probes should be placed, spacing probes at better intervals, widening probes under certain conditions, and Gaussian weighting the probe results. A more sophisticated algorithm, "Table Feline," described in [10], also better approximates the slope and length of the ellipse's major and minor axes. Both versions of Feline require just a few additional computations over previous algorithms.

In this paper, we first discuss previous work, including the best efficient software technique, and shortcomings of recent hardware anisotropic filtering techniques. We next describe the desired computations for using several probes along a line, show how to make these computations amenable to hardware, and discuss techniques to reduce the number of probes per pixel. Finally, we present several pictures comparing the various methods of filtering. More details about Feline can be found in [10].

2 PREVIOUS WORK

We first describe Elliptical Weighted Average (EWA), the most efficient direct convolution method known for computing a textured pixel. This provides a quality benchmark against which to compare other techniques. (We do not describe previous software efforts like [2] and [3], as we feel that EWA either supersedes these algorithms, or that they are so slow as to be in a different class.) We discuss trilinear filtering, which is popular but blurry. We delve more deeply into Texram, a chip that performs anisotropic filtering by repeated applications of an isotropic filter along a line, and discuss its weaknesses. We briefly mention other algorithms apparently similar to Texram, but which are not described in sufficient detail to analyze.

2.1 Elliptical Weighted Average

Paul Heckbert's and Ned Greene's Elliptical Weighted Average (EWA) algorithm [4][6] exactly computes the size, shape, and orientation of an elliptical filter like the one shown in Figure 4. If the center of the filter in texture space is translated to (0, 0), then the filter in texture space can be characterized as:

$$d^2(u, v) = Au^2 + Buv + Cv^2$$

The value d^2 represents the distance squared from the center of the pixel when the texel position is mapped back into screen space. Thus, d^2 can index a table of weights that is unrelated to the affine projection, but depends only upon the pixel filter.

EWA determines d^2 for each texel in or near the elliptical footprint. Texels inside the footprint ($d^2 \le 1$) are sampled, weighted, and accumulated. The result is divided by the sum of the weights, which is the elliptical filter's volume in texture space.

Given the partial derivatives $\partial u/\partial x$, $\partial v/\partial x$, $\partial u/\partial y$, and $\partial v/\partial y$, which represent the rates of change of u and v in texture space relative to changes in x and y in screen space, the biquadratic coefficients for computing d^2 are:

$A_{nn} = (\partial v/\partial x)^2 + (\partial v/\partial y)^{2;}$
$B_{nn} = -2 * (\partial u/\partial x * \partial v/\partial x + \partial u/\partial y * \partial v/\partial y);$
$C_{nn} = (\partial u/\partial x)^2 + (\partial u/\partial y)^2;$
$F = A_{nn} * C_{nn} - B_{nn}^2/4;$
$A = A_{nn}/F;$
$B = B_{nn}/F;$
$C = C_{nn}/F;$

Pixels that map to a large area in texture space can be handled by using mip-maps [12], where each level of a mip-map is ½ the height and width of the previous level. Heckbert [6] suggests sampling from a single mip-map level in which the minor radius is between 1.5 and 3 texels, though he later implemented unpublished code in which the minor radius is between 2 and 4 texels, in order to avoid subtle artifacts.

Even using mip-maps, highly eccentric ellipses may encompass an unacceptably large area. This area can be limited by computing the ratio of the major radius to the minor radius, and if this ratio is too large, widening the minor axis of the ellipse and rederiving the coefficients A, B, and C. The combination of mip-maps and ellipse widening allows EWA to compute a textured pixel with a constant time bound.

Choosing a mip-map level and testing for very eccentric ellipses requires computing the major and minor radii of the ellipse:

$root = \text{sqrt}((A - C)^2 + B^2);$
$A' = (A + C - root)/2;$
$C' = (A + C + root)/2;$
$majorRadius = \text{sqrt}(1/A');$
$minorRadius = \text{sqrt}(1/C');$

Widening an ellipse requires seven multiplies, a square root, an inverse root, and a divide. These setup computations, plus logic to visit only texels in or near the ellipse and compute d^2, have thus far precluded hardware implementation of EWA.

The only complaint that can be leveled against EWA's visual quality is its choice of a Gaussian filter. Other filters produce sharper images without introducing more aliasing artifacts (see Wolberg [13] for an excellent discussion). However, these filters have a radius of two or three pixels, which increases the work required to compute a textured pixel by a factor of four or nine. And as Lansdale [8] points out, none of these filters are as mathematically tractable as the Gaussian for unifying the reconstruction filter and projected pixel filter (warped prefilter).

2.2 Trilinear Filtering

Trilinear filtering emphasizes simplicity and efficiency at the cost of visual quality. Rather than computing the shape of the projected filter footprint, it uses a square filter in texture space. By blending two 2 x 2 bilinear filters from adjacent mip-map levels, trilinear filtering approximates a circular filter of an arbitrary size. Figure 5 shows a trilinear filter that (poorly) approximates the EWA filter shown in Figure 4. The axis tick marks are spaced one texel apart, while the grid is spaced at ½ texel intervals. Strictly speaking, because it blends two 2 x 2 bilinear filtering operations, a trilinear filter samples a square area of 2^n x 2^n texels. However, most of the filter volume resides inside a circle with the nominal filter radius. In the 2D pictures below, we thus show a trilinear filter's footprint as a circle of the nominal radius.

A trilinear filter blurs or aliases textures applied to surfaces that are obliquely angled away from the viewer. These artifacts arise because the fixed shape of the trilinear filter poorly matches the desired filter footprint, and so the trilinear filter samples data outside the ellipse, doesn't sample data inside the ellipse, or both.

2.3 Texram

Texram [11] provides higher visual quality than trilinear filtering with less complexity than EWA. Texram uses a series of trilinear filter probes along a line that approximates the length and slope of the major axis of EWA's elliptical footprint.

The Texram authors considered computation of the ellipse parameters too costly for hardware, and so substituted simplified approximations. These approximations underestimate the length of the major axis of the ellipse, introducing aliasing; overestimate the length of the minor axis, introducing blurring; and deviate from the slope of the major axis, introducing yet more blurring and aliasing. Nonetheless, with the exception of environment mapping, these errors are visually insignificant under typical perspective projections, as discussed further in Section 3.2 below.

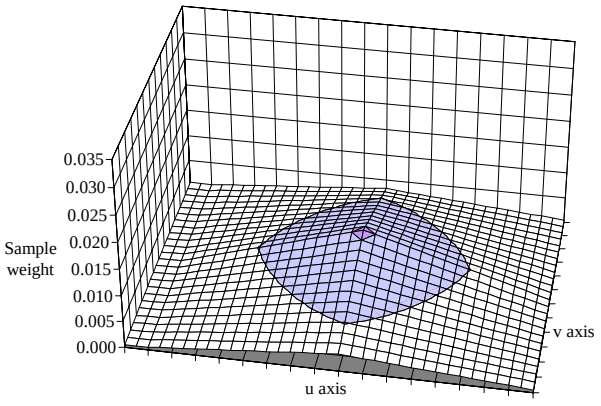

Figure 5: A trilinear filter approximation to Figure 4.

Texram has other problems that manifest themselves as aliasing artifacts. Its sampling line is usually much shorter than the ellipse, and the trilinear probes can be spaced too far apart. Texram always uses 2^n equally weighted probes, which causes poor high-frequency rejection along the major axis. These problems make Texram's visual quality noticeably inferior to EWA.

Texram uses the four partial derivatives to create two vectors in texture space: ($\partial u/\partial x$, $\partial v/\partial x$) and ($\partial u/\partial y$, $\partial v/\partial y$). The authors claim to sample roughly the area inside the parallelogram formed by these two vectors, by probing along a line that has the length and slope of the longer of the two vectors. This line can deviate from the slope of the major axis of EWA's elliptical filter by as much as 45°. This is not as bad as it sounds. The largest angular errors are associated with nearly circular filters, which are relatively insensitive to such errors in orientation.

Texram's sampling line can be shorter than the true ellipse's major axis by nearly a factor of four. One factor of two comes from Texram's use of the length of the longer vector as the length of the sample line. Note that if orthogonal vectors are plugged into the ellipse equations in Section 2.1 above, the major radius is the length of the longer vector, and so the ellipse's major diameter is actually *twice* the length of this vector. Texram's error is apparently due to an older paper by Paul Heckbert [5], in which he suggested using a filter diameter that is really a filter radius.

Another factor of two comes from non-orthogonal vectors. If the two vectors are nearly parallel and equal in length, the elliptical footprint is very narrow and has a major radius nearly twice the length of either vector. Again, this is not as bad as it sounds: typical perspective distortions yield a true ellipse radius that is no larger than about 7% of the longer vector.

Texram approximates the radius of the minor axis of the ellipse by choosing the shortest of the two parallelogram side vectors and the two parallelogram diagonals ($\partial u/\partial x + \partial u/\partial y$, $\partial v/\partial x + \partial v/\partial y$) and ($\partial u/\partial x - \partial u/\partial y$, $\partial v/\partial x - \partial v/\partial y$). If the side vectors are nearly parallel and the shorter is half the length of the longer, this approximation can be too wide by an arbitrarily large factor.

One of the Texram authors was unsure which values round up or down in the division that computes the number of probes. We have assumed values in the half-open interval [1.0 to 1.5) round to one probe, values in [1.5 to 3) round to two probes, values in [3 to 6) round to four probes, etc. Texram does not adjust the probe diameter when it rounds down (as discussed in Section 3.1 below), and so can space probes too far apart. Rather than the smoothly sloped "shield volcano" filter of EWA, Texram can use a "mountain range" filter with individual peaks. These peaks beat against repeated texture patterns to create phantom patterns.

Figure 6 shows an extreme example of these errors, in which ($\partial u/\partial x$, $\partial v/\partial x$) is (13, 0) and ($\partial u/\partial y$, $\partial v/\partial y$) is (12, 5). The area sampled by EWA is shown as the large heavily outlined ellipse, while Texram's trilinear filter footprints are shown as circles.

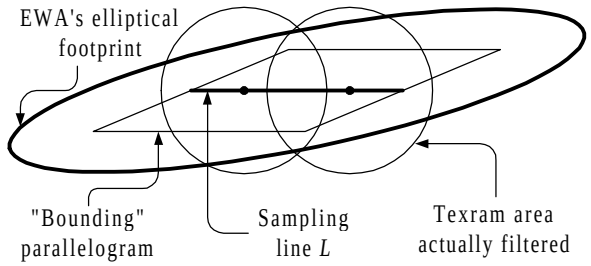

Figure 6: Texram area sampled vs. EWA.

2.4 Other Hardware Algorithms

Microsoft's Talisman [1] uses a filtering algorithm "in the spirit of" Texram. Few details are provided, but the aliasing evident in the examples suggest that they may have inherited some or all of Texram's problems. Evans & Sutherland holds U.S Patent #5,651,104 for using space-invariant probes along a line. The patent doesn't describe how to compute the probe line, but the diagrams imply a line that is at most a single pixel in length in screen space, which is once again so short that it will produce visible aliasing artifacts.

3 THE FELINE ALGORITHM

Like Texram, Feline uses several isotropic probes along a line *L* to implement an anisotropic filter. However, we compute a more appropriate length for the sampling line *L*, allow the number of probes to be any integer, don't space probes too far apart, and weight the probes using a Gaussian curve. Feline achieves higher visual quality than Texram with little additional logic.

We first describe the desired computations to yield the locations and weights for a series of probe points along a line. We then describe "Simple Feline," which inherits Texram's approximations of the major and minor radii, after which it implements the desired computations in a fashion suitable for hardware. Under highly distorted perspective projections, which may occur when environment mapping, Simple Feline's major and minor radii approximations result in blurring. "Table Feline," described in [10], uses a table to compute the ellipse axes more accurately. We conclude with techniques to reduce the number of probes, without substantially decreasing Feline's image quality.

3.1 The Desired Computations

The combination of multiple isotropic probes should closely match the shape of the EWA filter. Thus, the probe points should occur along the major axis of the ellipse, the probes should be Gaussian weighted, and the probe filter width should be equal to the minor axis of the ellipse.

(Theoretically, the probe filter width should be related to the width of the ellipse at each probe position. We initially did not investigate this because we didn't know how to optimize the trade-off between the probe diameter, probe weighting, probe spacing, and the number of probes. After implementing constant diameter probes, we saw no reason to pursue variable diameter probes. The "improvement" was unlikely to be visible, but would significantly increase the number of probes due to closer spacing of small probes near the ends of the ellipse.)

We compute *majorRadius* and *minorRadius* as in Section 2.1 above, and then the angle *theta* of the major axis:

theta = arctan(*B*/(*A*-*C*))/2;
// If *theta* is angle of minor axis, make it angle of major axis
if (*A* > *C*) *theta* = *theta* + π/2;

If *minorRadius* is less than one pixel (that is, we are magnifying along the minor axis, and possibly along the major axis), the appropriate radii should be widened—there is no point in making several probes to nearly identical locations. Heckbert's Master's Thesis [6] elegantly addresses this situation. He unifies the reconstruction and warped prefilter by using the following computations for *A* and *C* rather than the ones shown in Section 2.1 above:

$A_{nn} = (\partial v/\partial x)^2 + (\partial v/\partial y)^2 + 1;$
$C_{nn} = (\partial u/\partial x)^2 + (\partial u/\partial y)^2 + 1;$

This makes the filter radius sqrt(2) texels for a one-to-one mapping of texels into pixels. (The radius approaches one texel as magnification increases.) While theoretically superior, this wider filter blurs more than the radius one trilinear filter conventionally used for unity mappings and magnifications. In order to match this convention, and to make hardware implementation feasible, we clamp the radii to a minimum of one texel:

minorRadius = max(*minorRadius*, 1);
majorRadius = max(*majorRadius*, 1);

The space-invariant probes along the major axis have a nominal radius equal to *minorRadius*, and so the distance between probes should also be *minorRadius*. The end probes should be set in from the ellipse by a distance of *minorRadius* as well, so that they don't sample data off the ends of the ellipse. Therefore, the number of probes we'd like (*fProbes*), and its integer counterpart (*iProbes*), are derived from the ratio of the major and minor radii of the ellipse as follows:

fProbes = 2*(*majorRadius*/*minorRadius*) – 1;
iProbes = floor(*fProbes* + 0.5);
if (*iProbes* > *maxProbes*) *iProbes* = *maxProbes*;

To guarantee that texturing a pixel occurs in a bounded time, we clamp *iProbes* to a programmable value *maxProbes*. An application can use a small degree of anisotropy at high frame rates, and then allow more eccentric filters for higher visual quality when motion ceases.

When *iProbes* > *fProbes*, because *fProbes* is rounded up, we space probes closer than their radius, rather than blur the image by sampling data off the ends of the ellipse.

When *iProbes* < *fProbes*, either because *fProbes* is rounded down, or because *iProbes* is clamped, the ellipse will be probed at fewer points than desired. Spacing the probes farther apart or shortening the line *L* may cause aliasing artifacts. Instead, we blur the image by increasing *minorRadius* to widen the ellipse. Increasing *minorRadius* increases the level of detail and thus the nominal radius of the probe filter.

if (*iProbes* < *fProbes*)
    minorRadius = 2**majorRadius* / (*iProbes*+1);
levelOfDetail = $\log_2$(*minorRadius*);

Analogous to clamping *minorRadius* and *majorRadius*, we use a single probe in the smallest 1 x 1 mip-map, which reduces cycles spent displaying a repeated texture in the distance. We don't bother with a similar optimization for the 2 x 2 or 4 x 4 mip-maps. Consider the worst 2 x 2 case, in which a checkerboard is mirror repeated, and an ellipse with a *minorRadius* of 1 is centered at a corner of the texture map. Figure 7 depicts this situation, where the thin lines delineate texels, and the thick lines delineate the (repeated) 2 x 2 mip-map. The circle on the left uses one probe to compute an all-white pixel. The ellipse on the right uses 6 probes to compute the darkest possible pixel of 52% white, 48% shaded. (The white texels apparently inside the ends of the ellipse don't contribute to the pixel's color, as only texel centers are sampled.) Since longer ellipses converge so slowly to an intermediate color, we restrict ourselves to the trivial adjustment:

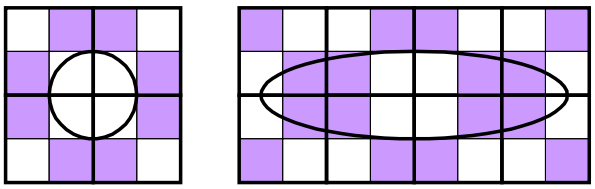

Figure 7: Ellipses in a 2 x 2 texture map oscillate around a blend of the two colors as eccentricity increases.

```
if (levelOfDetail > texture.maxLevelOfDetail) {
    levelOfDetail = texture.maxLevelOfDetail;
    iProbes = 1;
}
```

We compute the stepping vector $(\Delta u, \Delta v)$, which is the distance between each probe point along the line, as follows:

$lineLength = 2*(majorRadius - minorRadius)$;
$\Delta u = \cos(theta) * lineLength / (iProbes - 1)$;
$\Delta v = \sin(theta) * lineLength / (iProbes - 1)$;

(The stepping vector is irrelevant if *iProbes* is 1.) The sample points are distributed symmetrically about the midpoint (u_m, v_m) of the sampling line L in the pattern:

$$(u_n, v_n) = (u_m, v_m) + n/2 * (\Delta u, \Delta v)$$

where $n = \pm1, \pm3, \pm5, \ldots$ if *iProbes* is even, as shown in Figure 8, and $n = 0, \pm2, \pm4, \ldots$ if *iProbes* is odd, as shown in Figure 9.

We apply a Gaussian weight to each probe n by computing the distance squared of the probe from the center of the pixel filter in screen space, then exponentiating:

$d = n/2 * \text{sqrt}(\Delta u^2 + \Delta v^2) / majorRadius$;
$d^2 = n^2/4 * (\Delta u^2 + \Delta v^2) / majorRadius^2$;
$relativeWeight = e^{-\alpha * d^2}$;

Finally, we divide the accumulated probe results by the sum of all the weights applied.

3.2 Implementing Simple Feline

Simple Feline implements the above computations, except it uses Texram's ellipse axes approximations rather than computing the exact values. We use the longer of the two vectors $(\partial u/\partial x, \partial v/\partial x)$ and $(\partial u/\partial y, \partial v/\partial y)$ as the major radius, and the shortest of those and the two diagonals $(\partial u/\partial x + \partial u/\partial y, \partial v/\partial x + \partial v/\partial y)$ and $(\partial u/\partial x - \partial u/\partial y, \partial v/\partial x - \partial v/\partial y)$ as the minor radius length.

We were surprised that these approximations work essentially as well as the exact values under typical perspective projections. We discovered that the two vectors $(\partial u/\partial x, \partial v/\partial x)$ and $(\partial u/\partial y, \partial v/\partial y)$ are more or less orthogonal under typical perspective distortions. In the images shown below, the angle between the two are in the range 90° ± 30°, and the most extreme angles occur with very unequal vector lengths. The simple approximations are tolerably close to the true values under these conditions.

We use a two-part linear approximation for the vector length square root. Without loss of generality, for a vector (a, b) assume that $a, b > 0$ and $a > b$. The following function is within ±1.2% of the true length $\text{sqrt}(a^2 + b^2)$:

```
if (b < 3a/8) return a + 5b/32
else return 109a/128 + 35b/64
```

We do not compute the stepping vector with trigonometric functions, but instead scale the longer vector directly. Call the longer vector components (*majorU*, *majorV*). Either this vector describes *majorRadius*, or else *iProbes* is one and the stepping vector is irrelevant. By substituting *majorU*/*majorRadius* for cosine, and *majorV*/*majorRadius* for sine, we get:

$r = minorRadius / majorRadius$;
$i = oneOverNMinusOneTable[iProbes]$;
$\Delta u = 2*(majorU - majorU*r) * i$;
$\Delta v = 2*(majorV - majorV*r) * i$;

Finally, we use a triangularish two-dimensional weight table to avoid computing and exponentiating d^2. We use the smaller of *fProbes* truncated to a couple fractional bits, or *iProbes*, as the weight table's row index, so that each row of weights applies to a small range of ellipses. The column index is floor((abs(*n*)+1)/2). By dividing each of the relative weights in a row by the sum of the weights for that row, the weights in each row sum to 1. Consequently, we need not normalize the final accumulated result. Note that if *iProbes* is odd, the W_0 entry in a row should count half as much as the other entries when computing the sum: it is used once, while the other weights are used twice.

Most of the computations specific to Feline can use group scaled numbers with a precision of 8 bits. (The center point (u_m, v_m) must still be computed with high precision, of course.) Small errors cause sampling along a line at a slightly different angle, and at intervals that are slightly smaller or larger than desired. These arithmetic errors are negligible compared to the inaccuracies caused by the gross approximations to the ellipse axes.

3.3 Increasing Efficiency

We investigated how far we could "push the envelope" to reduce the number of probes by shortening and widening the ellipse, and by spreading probe points farther apart than their radius.

We can shorten the ellipse using a *lengthFactor* <= 1:

```
majorRadius = max(majorRadius * lengthFactor,
                  minorRadius);
majorU *= lengthFactor;
majorV *= lengthFactor;
```

The code in Section 3.1 proportionately widens an ellipse more when rounding down a small value of *fProbes* than a large one. We can instead compute *iProbes* so that for all values of *fProbes*, we widen the ellipse to at most a *blurFactor* times the minor radius. We also allow stretching the distance between probe positions by up to *aliasFactor* times the probe filter radius:

$f = 1 / (blurFactor * aliasFactor)$;
$iProbes = \text{ceiling}(f * 2 * (majorRadius/minorRadius)) - 1$;

If *iProbes* is not clamped to *maxProbes*, we blur (widen the ellipse) by increasing *minorRadius* by up to *blurFactor*:

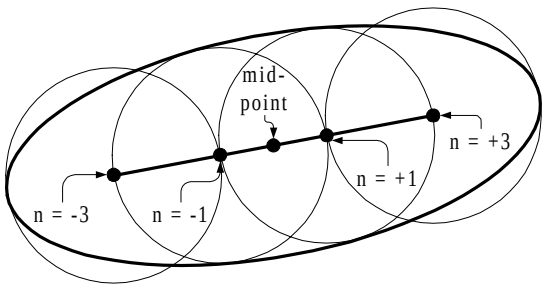

Figure 8: Positioning an even number of probes.

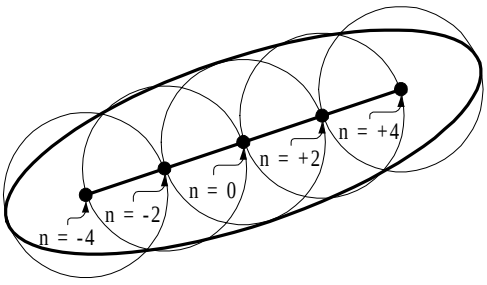

Figure 9: Positioning an odd number of probes.

minorRadius = min(2**majorRadius* / (*iProbes*+1),
minorRadius * *blurFactor*)

The computations of Δu and Δv automatically make up any remaining difference between *iProbes* and *fProbes* by increasing probe spacing. If *iProbes* is clamped, we blur (in excess of *blurFactor*) to the point where the computations of Δu and Δv will increase probe spacing by *aliasFactor*:

minorRadius = 2 * *majorRadius* /
((*iProbes*+1) * *aliasFactor*);

We chose two sets of parameter values empirically. The "high-quality" set (*lengthFactor* 0.97, *blurFactor* 1.16, *aliasFactor* 1.15) reduces the number of probes by 24% with almost no degradation of image quality, compared to the constant rounding of Section 3.1. The "high-efficiency" set (*lengthFactor* 0.97, *blurFactor* 1.31, *aliasFactor* 1.36) uses the same number of probes as Texram to provide images that contain more artifacts than the "high quality" setting, but are nonetheless much better than Texram.

The high-efficiency *aliasFactor* creates large valleys between the peaks of a trilinear filter, especially along diagonal probe lines. We obtained slightly better images by changing the probe filter from a bilinear filter on each of the two adjacent mip-map levels to a Gaussian filter truncated to a 2 x 2 square. We then linearly combine the two Gaussian results using the fractional bits of the level of detail. (This also makes single-probe magnifications look better.) A hardware trilinear filter tree is easily adapted to implement Gaussian rather than bilinear weightings [9]. Four copies of a small one-dimensional table map the fractional bits of *u* and *v* on each of the two mip-maps to Gaussian weights.

4 COMPARISONS WITH PREVIOUS WORK

Figure 10 through Figure 14 show various algorithms generating a pattern of curved lines. Figure 15 through Figure 18 show a floor of bricks, and Figure 19 through Figure 22 show magnified texture-mapped text. Texram images use the original algorithm; correcting the errors described in Section 2.3 above results in many more probes *and* degrades visual quality! Aliasing artifacts mostly remain, and images significantly blur due to the equal weighting of probes. Simple Feline images use parameters as described in Section 3.3 above, and a mip-mapped Gaussian for the probe filter. Mip-mapped EWA samples from a mip-map level where the minor radius is between 1.5 and 3 texels; this looks identical to a radius between 2 and 4, but samples about half as many texels. Trilinear, Texram, and Feline images use a radius 3 Lanczos filter to create mip-maps. EWA images use a box filter: the Lanczos filter causes "bluriness banding" artifacts when EWA jumps from using a large ellipse in one mip-map to using a small ellipse in the next.higher mip-map.

Feline with high-quality parameters generates images comparable to EWA, but with slightly stronger Moiré patterns. The only exception occurs if a box filter is used to create mip-maps for textures like checkerboards. Because the base texture and all its

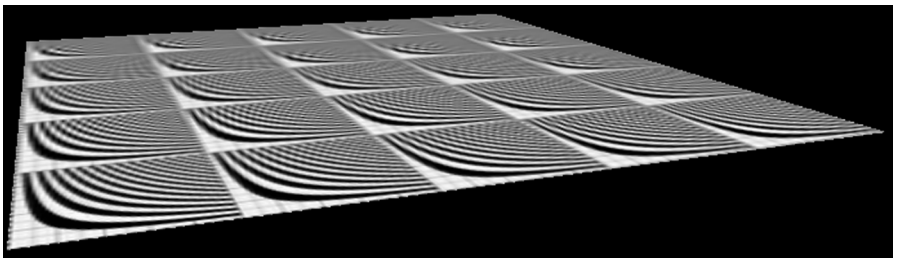

Figure 10: Trilinear paints curved lines with blurring.

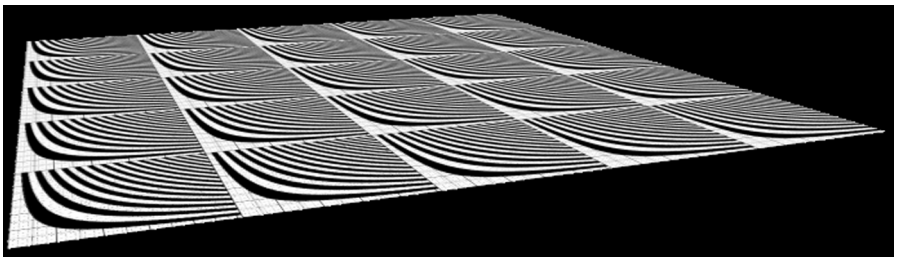

Figure 12: Texram paints curved lines with strong Moiré artifacts.

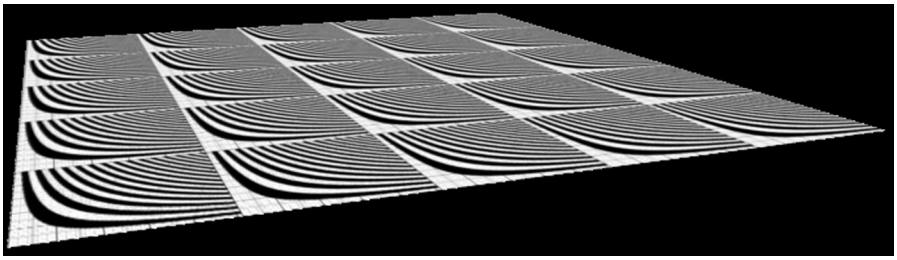

Figure 11: "High-efficiency" Simple Feline paints curved lines with fewer artifacts.

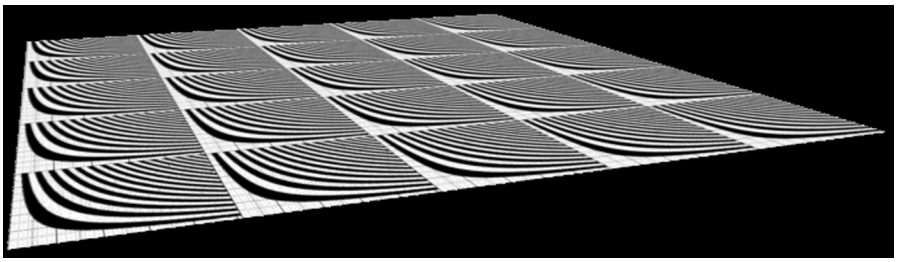

Figure 13: "High-quality" Simple Feline paints curved lines with few artifacts.

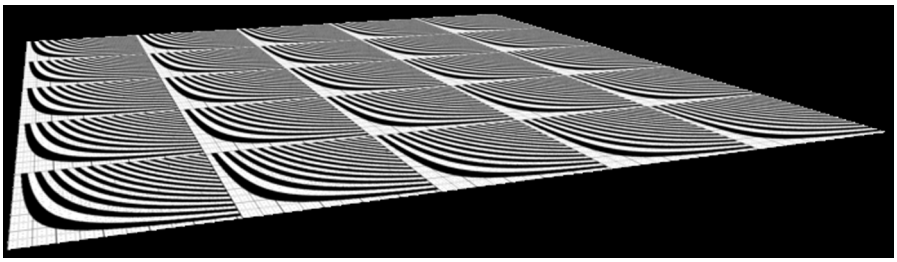

Figure 14: Mip-mapped EWA paints curved lines with few artifacts.

mip-maps then contain illegally high frequencies that Feline's relatively narrow filter cannot remove, Feline displays much stronger Moiré artifacts than EWA. Using a better filter, such as the Lanczos, to create the mip-maps makes Feline display *fewer* artifacts than EWA—Feline is more likely to use filtered mip-mapped data, rather than the unfiltered base texture.

Both sets of Feline images are much sharper, and exhibit far fewer Moiré artifacts, than those generated by trilinear filtering. Though not shown here, we note that high-efficiency Feline and Texram are both subject to "probe banding" on repeated textures: some images show a visible line where the number of probes increases from one value to another.

Texram images sometimes seem a little sharper than Feline images, but then, aliased images always seem sharper than antialiased images. Repeated texture patterns amplify Texram's aliasing problems to create strong Moiré patterns, as shown in the curved lines and bricks images. These patterns are even more disturbing in moving images, where they shimmer across the surface. Texram's aliasing is more subtle in non-repeated textures, such as text. Comparing the high-efficiency Feline images to Texram is especially interesting: both use the same number of probes, but the Feline images exhibit far fewer artifacts. Experiments show that Feline's quality is due to the use of a Gaussian probe filter, the Gaussian weighting of probe results, and the end-to-end coverage of the ellipse.

Higher visual quality comes at increased computational cost for setup and sampling. But much of Feline's setup can be performed in parallel with the perspective divide pipeline, and so increases pipeline length over Texram by only a few stages. Feline's setup costs are substantially smaller than mip-mapped EWA's.

Both Feline and Texram access eight texels each probe, and probes overlap substantially (especially in the smaller of the two mip-maps). A texel cache [7][9] eliminates most redundant memory fetches. We assume these algorithms can perform one probe per cycle; higher performance requires duplicating large portions (100k to 200k gates) of the texture mapping logic.

Mip-mapped EWA doesn't fetch texels more than once per pixel and samples a substantially larger area. "Optimistic EWA" naively assumes we can sample 8 texels/cycle on all but the last cycle for each ellipse. "Realistic EWA" assumes that hardware traverses the ellipse using a 4 x 2 texel "stamp" for *u*-major ellipses, and a 2 x 4 stamp for *v*-major ellipses. Thus, each cycle several of the stamp's texels usually lie outside the ellipse.

Figure 23 shows how many cycles/pixel each algorithm uses for different viewing angles of one exemplary surface. At 0°, the surface normal is parallel to the viewing angle, and mip-mapped

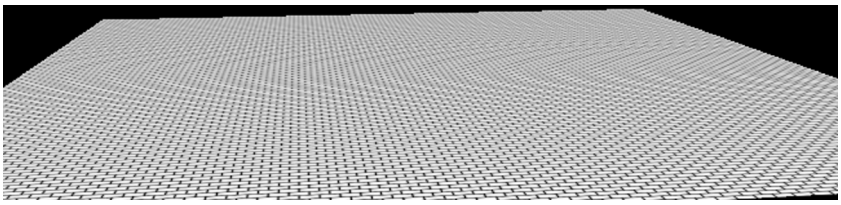

Figure 15: Texram paints bricks with herringbone artifacts.

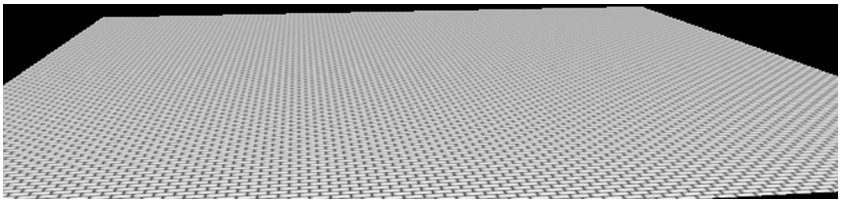

Figure 16: "High-efficiency" Simple Feline paints bricks with fewer artifacts.

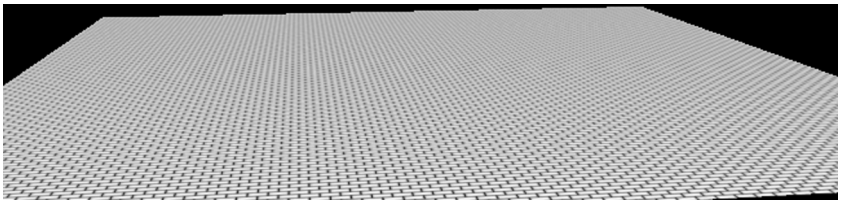

Figure 17: "High-quality" Simple Feline paints bricks with few artifacts.

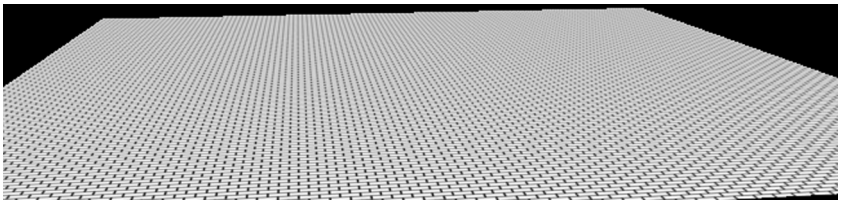

Figure 18: Mip-mapped EWA paints bricks with fewest artifacts.

Figure 19: Trilinear paints blurry text.

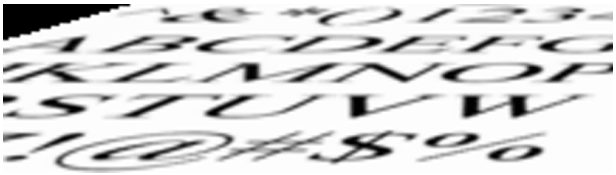

Figure 21: "High-efficiency" Simple Feline paints smooth text.

Figure 20: Texram paints text with stairstepping.

Figure 22: Mip-mapped EWA paints smooth text.

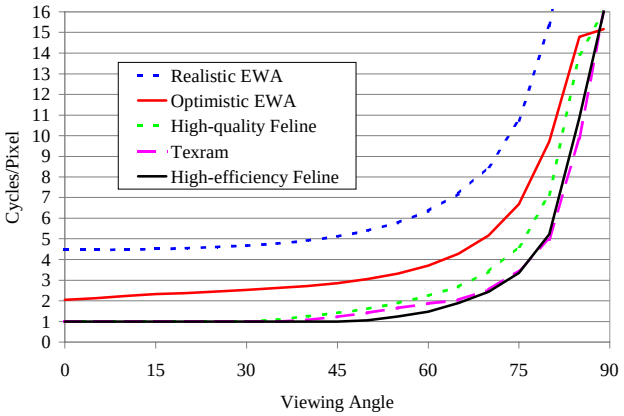

Figure 23: Performance at increasingly oblique viewing angles.

EWA samples the same size circle for each pixel. We made this circle's area the same as would be obtained by averaging results from randomly distributed viewing distances. This graph should be interpreted like EPA gas mileage numbers: it is useful for relative comparisons, but mileage will vary depending upon position on the screen, perspective distortion, etc.

Finally, note that if a scene uses multiple textures per surface, anisotropic texture mapping performance doesn't always slow down by these cycles/pixel ratios. For example, illumination maps tend to be small, so are usually magnified [7], which takes a single probe. They also tend to be blurry (that is, contain mostly low frequencies), so even when minified, an application might limit illumination mapping to one or two probes per pixel.

5 CONCLUSIONS

Feline provides nearly the visual quality of EWA, but with much simpler setup and texel visiting logic, and many fewer cycles per textured pixel. Feline provides better image quality than Texram, especially for repeated textures, even when limited to use the same number of probes. Feline requires somewhat more setup and texel weighting logic than Texram, but this cost is small compared to the increase in visual quality. Feline can be built on top of an existing trilinear filter implementation; for better results, the trilinear filter can be converted to a mip-mapped Gaussian at little cost. Since several aspects of Feline are parameterized, Feline can gracefully degrade image quality in order to keep frame rates high during movement. This degradation might accentuate aliasing for irregular textures, in order to preserve image sharpness, and accentuate blurring for repeated regular textures, in order to avoid Moiré artifacts.

In the Sep/Oct 1998 issue of *IEEE Computer Graphics and Applications*, Jim Blinn wrote in his column that "No one will ever figure out how to quickly render legible antialiased text in perspective. Textures in perspective will always be either too fuzzy or too jaggy. No one will ever build texture-mapping hardware that uses a 4x4 interpolation kernel or anisotropic filtering." Feline is simple enough to implement, yet of high enough visual quality, to prove him at least partially wrong.

6 ACKNOWLEDGEMENTS

Thanks to Paul Heckbert for answering questions and for providing us with the EWA source code, and to Gunter Knittel for answering questions about Texram.

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